

## VARIATIONS OF THE FOUR CUBE PROBLEM

By Michael O Hinder

The four cube problem has been around for over a century under various names such as Katzenjammer, Great Tantalizer, and Instant Insanity. These puzzles have graph theoretic solutions that show that many of them have the same unique solution but with different colors. Unique solutions for the five and six cube problems are known as well. I created a program that can create and evaluate these types of puzzles. My research has shown that there exist cubes that make a solution impossible when they replace any cube in the five and six cube puzzles. There are many different non-isomorphic four, five, and six cube puzzles each with unique solutions that can be verified graphically. Some but not all of them have a no-solution cube as well. This list of puzzles is not exhaustive but shows that there is more than one way to have a unique solution for the five and six cube problems.

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by

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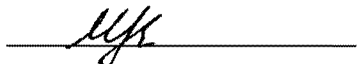
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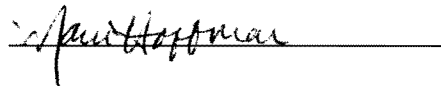
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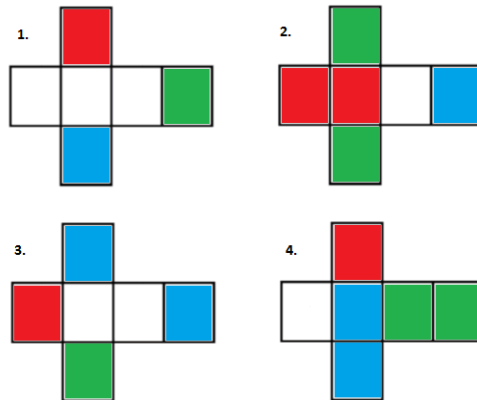
## Chapter I

### Introduction to the Four Cube Problem

The four cube problem has been around for over one hundred years. The Katzenjammer puzzle (Schossow U.S. Patent No. 646,463, 1900) was patented in 1900 and featured the four playing card symbols with the goal being to stack them such that each side of the stack had one of each symbol. Other puzzles would be created after this such as The Great Tantalizer and Instant Insanity (Stagmann, 2004). These two puzzles, along with many others, are isomorphic to the original Katzenjammer (Storer, 2013). While this puzzle can be solved using various techniques, graph theory students focus on its graph theoretic solutions. However, students have been able to solve the four cube puzzle without graph theory. Harder puzzles with more cubes needed to be created in order to challenge these students and to show how graph theory can help solve them in ways that traditional puzzle techniques cannot. To generate these harder puzzles I created a program that can instantly solve four, five, and six cube problems (Hinder, 2014). This program can also generate a cube that, when substituted for any cube in the puzzle, will make a solution impossible. When given all but one of the cubes in the puzzle it can also generate a cube that will make the puzzle have a unique solution. The limitation to this program is that it can only tell you if a cube exists but it cannot say a cube does not exist.

For our example we'll be using the Instant Insanity puzzle shown in Figure 1. This puzzle consists of four cubes whose faces are one of four colors: red, white, blue, and green. The object of the

game is to stack these cubes on top of one another such that each side of the stack has one face of each color.



*Figure 1.* The Instant Insanity Problem.

Before proceeding, a basic understanding of definitions is required. We will be defining our terms according to Chartrand & Zhang (2005).

Edge: A line connecting an unordered pair of vertices.

Graph: a finite, non-empty set of vertices and edges.

Multi-graph: a finite, non-empty set of vertices and a set of edges where every two vertices are joined by a finite number of edges where an edge is permitted to join a vertex to itself.

Subgraph: a subset of the vertices and edges of a graph such that if an edge is included, so are its endpoints.

Spanning Subgraph: a subgraph of graph  $G$  with the same vertex set as  $G$ .

Degree: the degree of a vertex is the number of edges incident with it.

2-Regular Graph: a graph where every vertex has degree 2.

A graph-theoretic solution can be found by labeling four vertices with “R”, “W”, “G”, and “B” to indicate the colors and then connecting an edge between two vertices if they are on opposite sides of the cube. We label the edges 1-4 to indicate which cubes contain that edge. There will be a total of 12 edges. Graphs can be displayed as either a multi-graph showing every edge or as a simplified composite graph as shown in Figure 2.

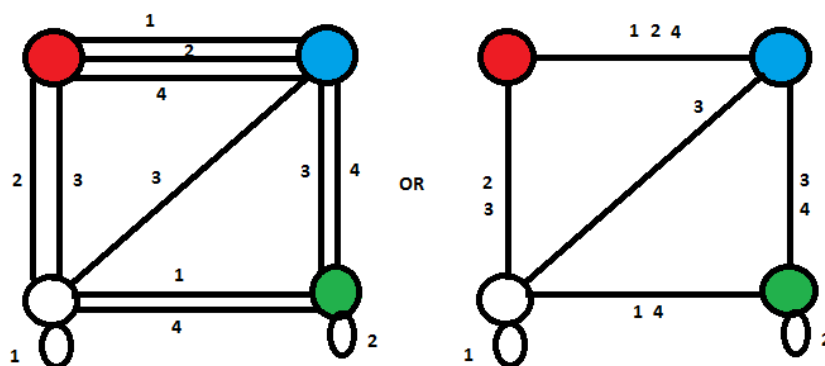


Figure 2. The multi-graph for Instant Insanity.

For our purposes we will be showing the composite graphs and subgraphs. To find every solution for this problem we need to consider all 2-regular spanning subgraphs of the composite graph where each subgraph contains one edge from each cube. Two subgraphs are considered a viable solution if they do not have a specific edge in common. One subgraph will represent the front and back and the other will represent the left and right sides. A common edge from the same cube between these subgraphs would suggest that the same cube is used in two different positions which is impossible. Since we use every color, each color is one degree, and we’re

looking at a pair of sides, each subgraph has to be a 2-regular spanning subgraph. Figure 3 contains a list of all possible 2-regular spanning subgraphs.

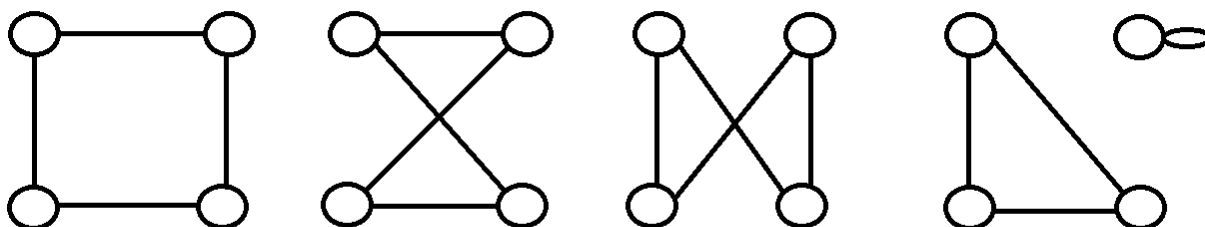


Figure 3.1. The 2-regular spanning subgraphs. Four of the 2-regular spanning subgraphs.

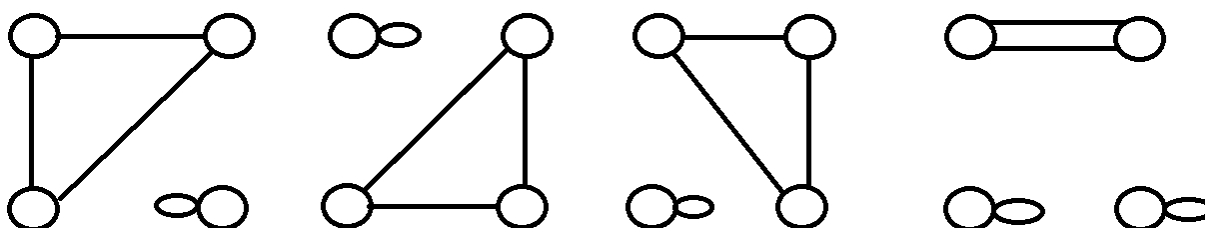


Figure 3.2. The 2-regular spanning subgraphs. Four of the 2-regular spanning subgraphs.

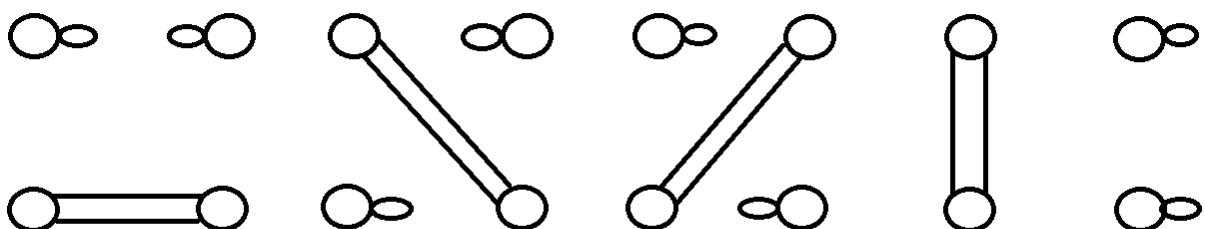


Figure 3.3. The 2-regular spanning subgraphs. Four of the 2-regular spanning subgraphs.

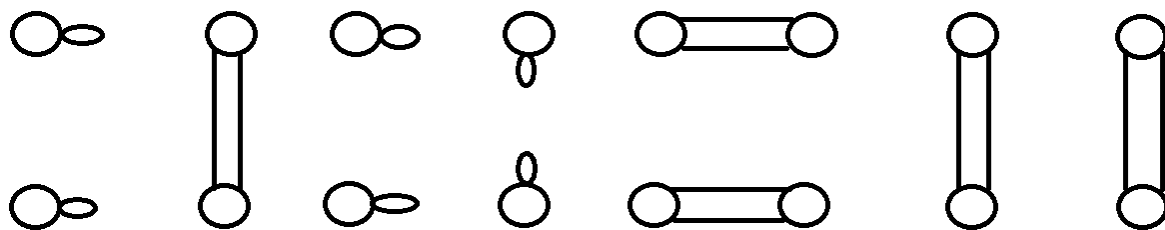


Figure 3.4. The 2-regular spanning subgraphs. Four of the 2-regular spanning subgraphs.

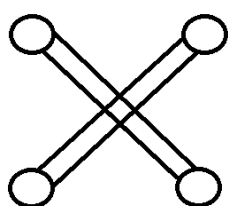


Figure 3.5. The 2-regular spanning subgraphs. The last of the 2-regular spanning subgraphs.

It's important to note that many of these graphs look similar. This is because they are the same graph only rotated. In future lists we will only show one of the rotations and the others will be accounted for but not shown. The three possible spanning subgraphs with one edge from each cube for this problem are shown in Figure 4. We found these by considering each of the seventeen subgraphs from Figure 3.

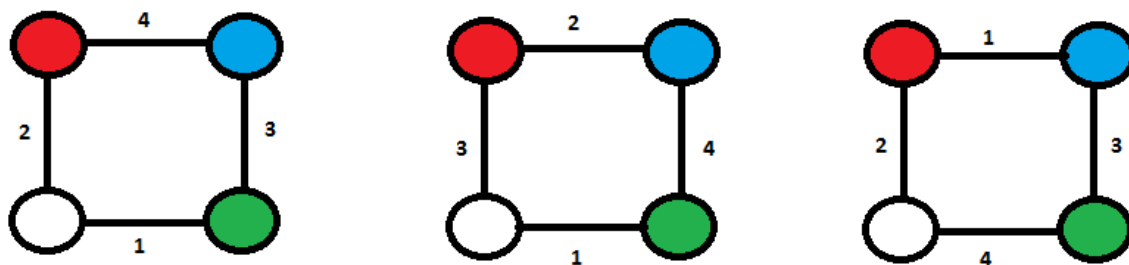


Figure 4. The potential solution subgraphs.

Subgraphs one and two have the 1 edge in common. Subgraphs 1 and 3 have the 2 edge in common. Subgraphs 2 and 3 have no edges in common and therefore represent the solution. Subgraph 2 will represent the front and back of the stack and graph 3 will represent the left and right sides. Because this is our only possible pair, this puzzle has a unique solution.

### **Interesting Observations**

The four cube problem shown in Figure 1 is isomorphic to Instant Insanity. The puzzle Enigmus can be created by taking three of these cubes and changing the fourth one (Stagmann, 2004). My program has constructed puzzles not isomorphic to Instant Insanity, Enigmus, Cubo Color Puzzle, or Psykonosis. There exists a fifth cube that will make finding a solution impossible when it replaces any of the cubes from Instant Insanity (Koencke & Winters, n.d.). The existence of this cube is already known and therefore it has been omitted from this paper.

## Chapter II

### The Six Choose Five Cube Problem

Similar to the four cube problem, a five cube problem can be created. In our example we will use five cubes with five colors such that every cube will have all of the colors and then one color repeated. There is a unique solution for the cubes we have selected. The five cubes are shown in Figure 5.

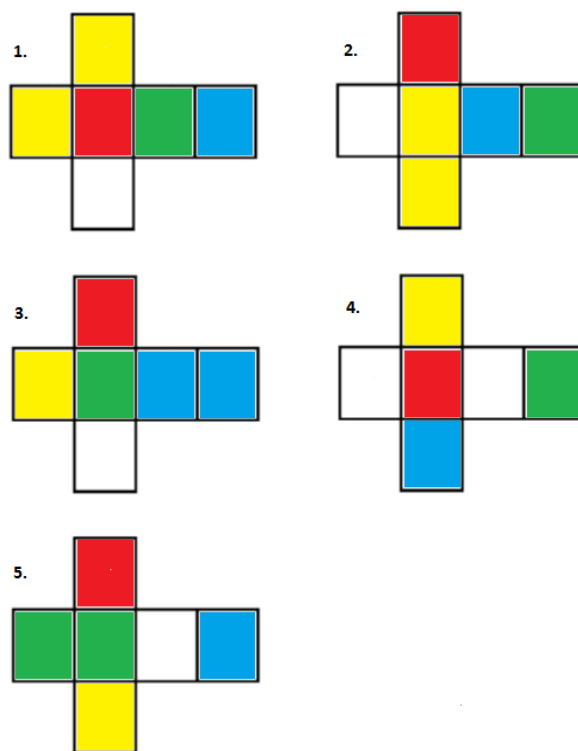
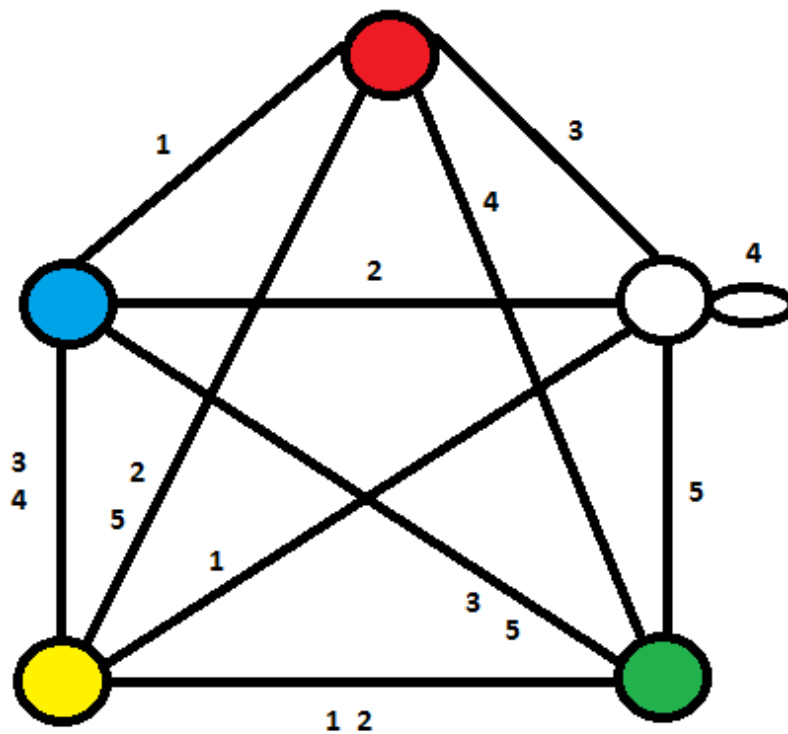


Figure 5. The five cube problem.

Similar to the previous problem we will create a multi-graph and then compare every possible two-regular spanning subgraph in order to find our solutions. As before, the multi-graph is created by connecting 2 colored vertices for every pair of opposite faces for every cube. The multi-graph for this puzzle is shown in figure 6.



*Figure 6.* The multi-graph for the five cube problem.

For the sake of space we will omit showing the rotations but they have been evaluated as well.

The possible 2-regular spanning subgraphs are shown in Figure 7.

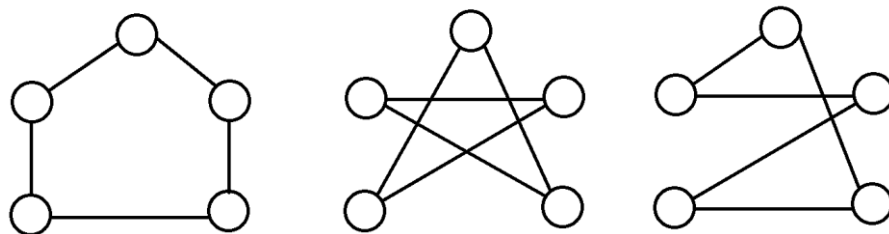


Figure 7.1. The 2-regular spanning subgraphs for the five cube problem. Three subgraphs.

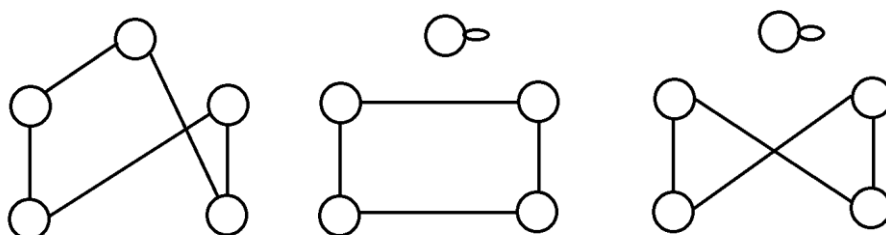


Figure 7.2. The 2-regular spanning subgraphs for the five cube problem. Three subgraphs.

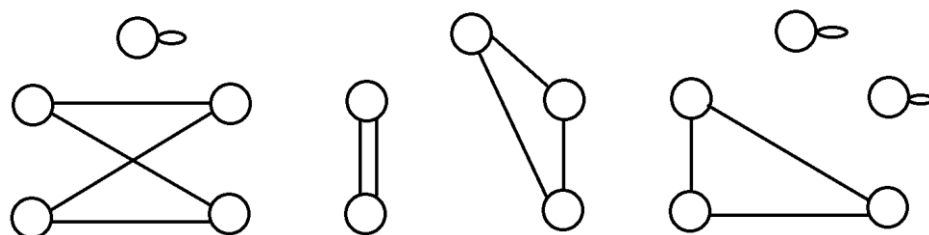


Figure 7.3. The 2-regular spanning subgraphs for the five cube problem. Three subgraphs.

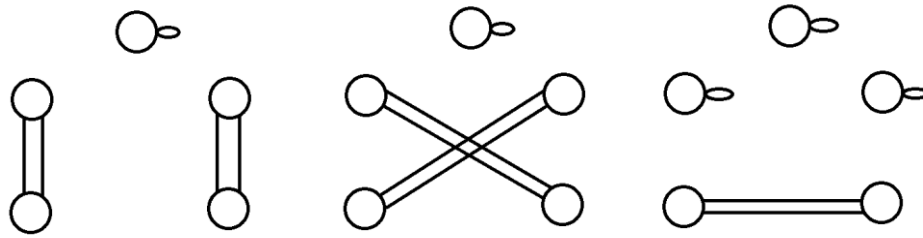


Figure 7.4. The 2-regular spanning subgraphs for the five cube problem. Three subgraphs.

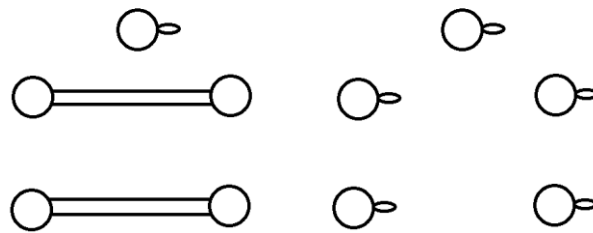


Figure 7.5. The 2-regular spanning subgraphs for the five cube problem. Two subgraphs.

This particular set of cubes results in three possible 2-regular subgraphs with one edge from each cube as shown in Figure 8.

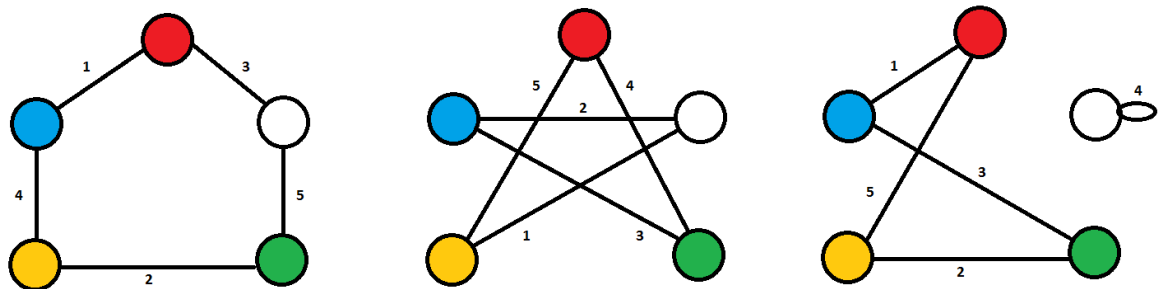
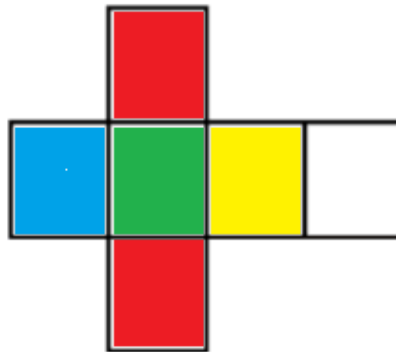


Figure 8. The 2-regular spanning subgraphs containing potential solutions.

The first two subgraphs share no edge in common and therefore they represent a solution. The first and third subgraphs share the edge labeled 1. The second and third subgraphs share the edge labeled 5. Therefore this puzzle has a unique solution.

### **The Six Choose Five No Solution Cube**

My program has shown that a sixth cube can be created that will yield no solutions for this set of cubes when substituted in for any cube. This no-solution cube follows the same rules as the others where no color is repeated more than once. The existence of this cube creates the six choose five problem. The multi-graph of this problem is shown in Figure 10.



*Figure 9.* The layout of the no-solution cube.

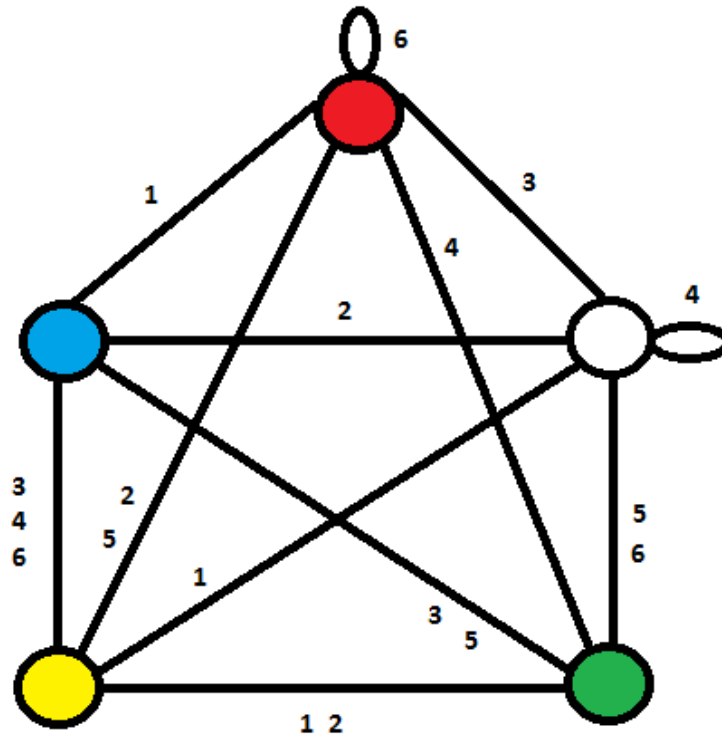
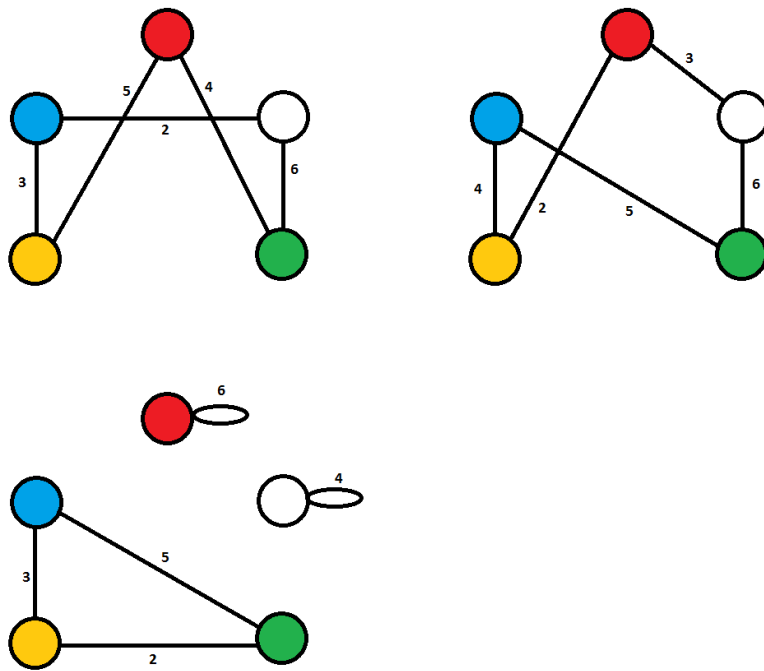


Figure 10. The multi-graph of the six choose five problem.

Since the number of cubes in the solution does not change when the sixth cube is introduced it replaces one of those five cubes. Subgraphs that have the same cube replaced are grouped together and compared. As shown in Figures 11-15, none of these pairs will yield a solution.



*Figure 11.* The subgraphs found by replacing the first cube with the no-solution cube. The first two subgraphs share edge 6. The last two subgraphs share edge 5. The first and last subgraphs share edge 3.

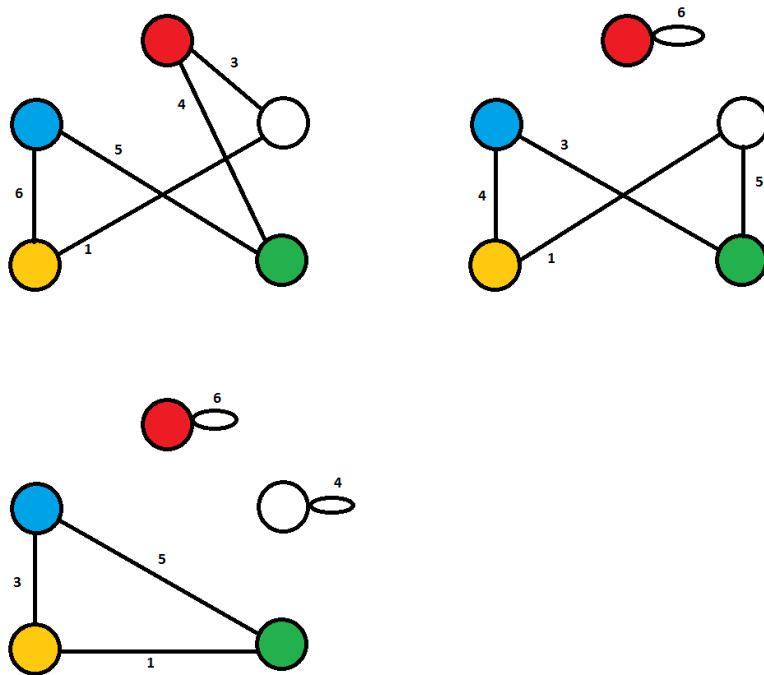


Figure 12. The subgraphs found by replacing the second cube with the no-solution cube. The first two subgraphs share edge 1. The last two subgraphs share edge 6. The first and last subgraphs share edge 5.

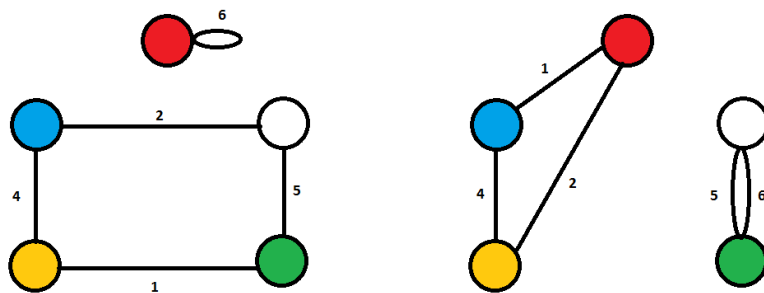


Figure 13. The subgraphs found by replacing the third cube with the no-solution cube. These subgraphs share edge 4.

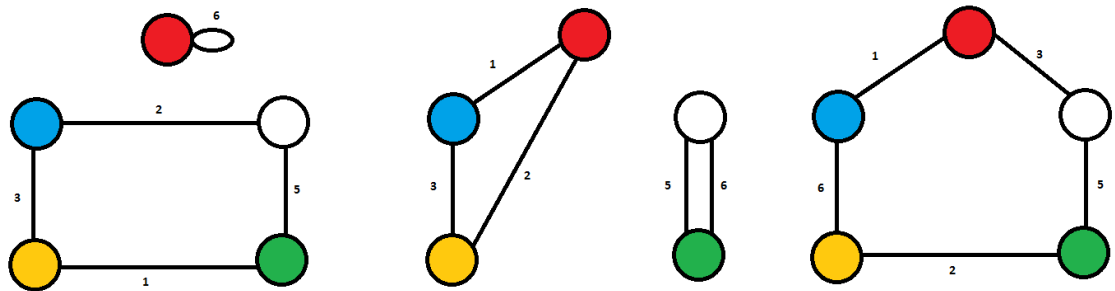


Figure 14. The subgraphs found by replacing the fourth cube with the no-solution cube. These subgraphs share edge 5.

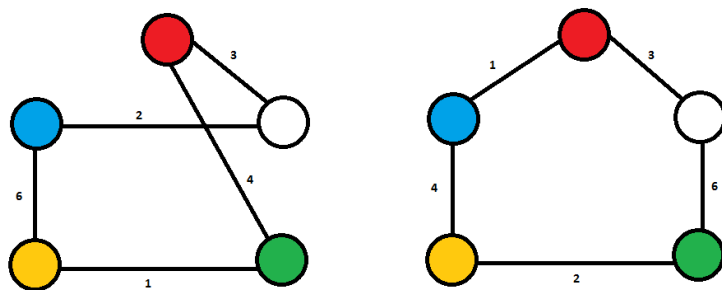


Figure 15. The subgraphs found by replacing the fifth cube with the no-solution cube. These subgraphs share edge 3.

One color is repeated in the no-solution cube. If that color is opposite itself, there will be a self-loop in the graph. Otherwise, that color appears twice, so there are two edges incident with that color in the graph which we will note as a double edge. We claim that this cube is the only no-solution cube for this puzzle. To prove this we will show every cube's possible subgraphs followed by their solutions. Some puzzles might have more than one solution but since we're looking for solutions in general we will only show one of them. To follow our color rules, the

self-loop composite graphs contain two other edges that do not share a vertex. Likewise, the double edge composite graphs have two edges meeting at their shared color vertex while the third edge is the only possible edge that doesn't share a vertex with the previous edges. There are three ways to draw a self-loop for every color and six ways to draw a double edge graph. Since there are five colors we will examine forty-five graphs per puzzle as shown in Figures 16-25.

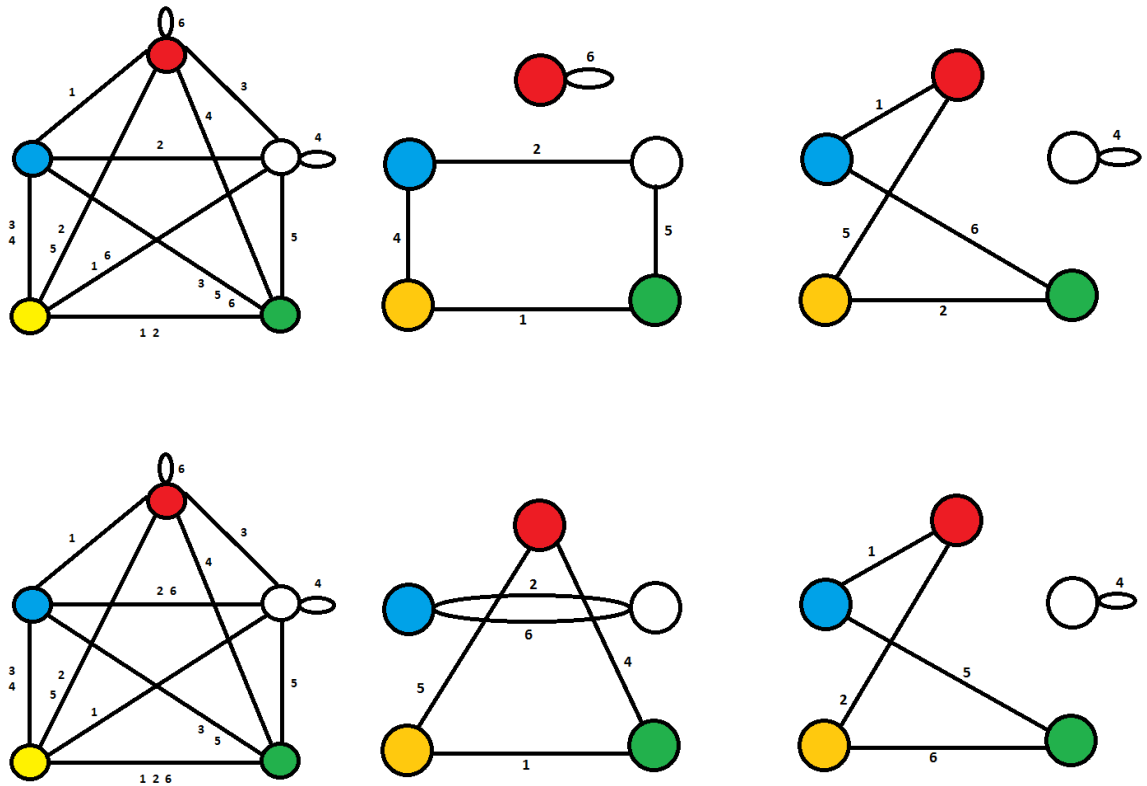


Figure 16. The multi-graphs and subgraph solutions from the self-loop on the red vertex.

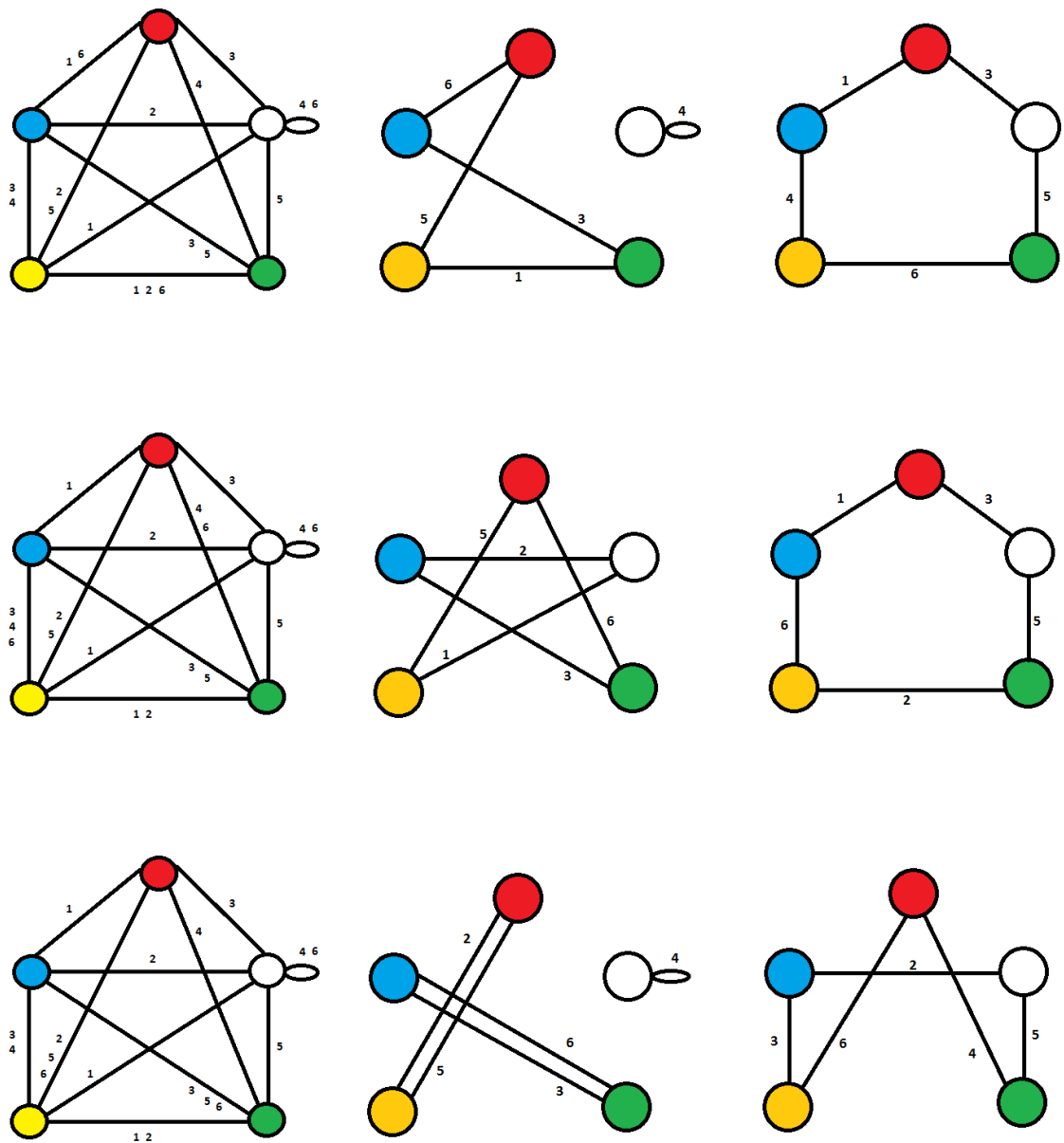


Figure 17. The multi-graphs and subgraph solutions of a self-loop on the white vertex.

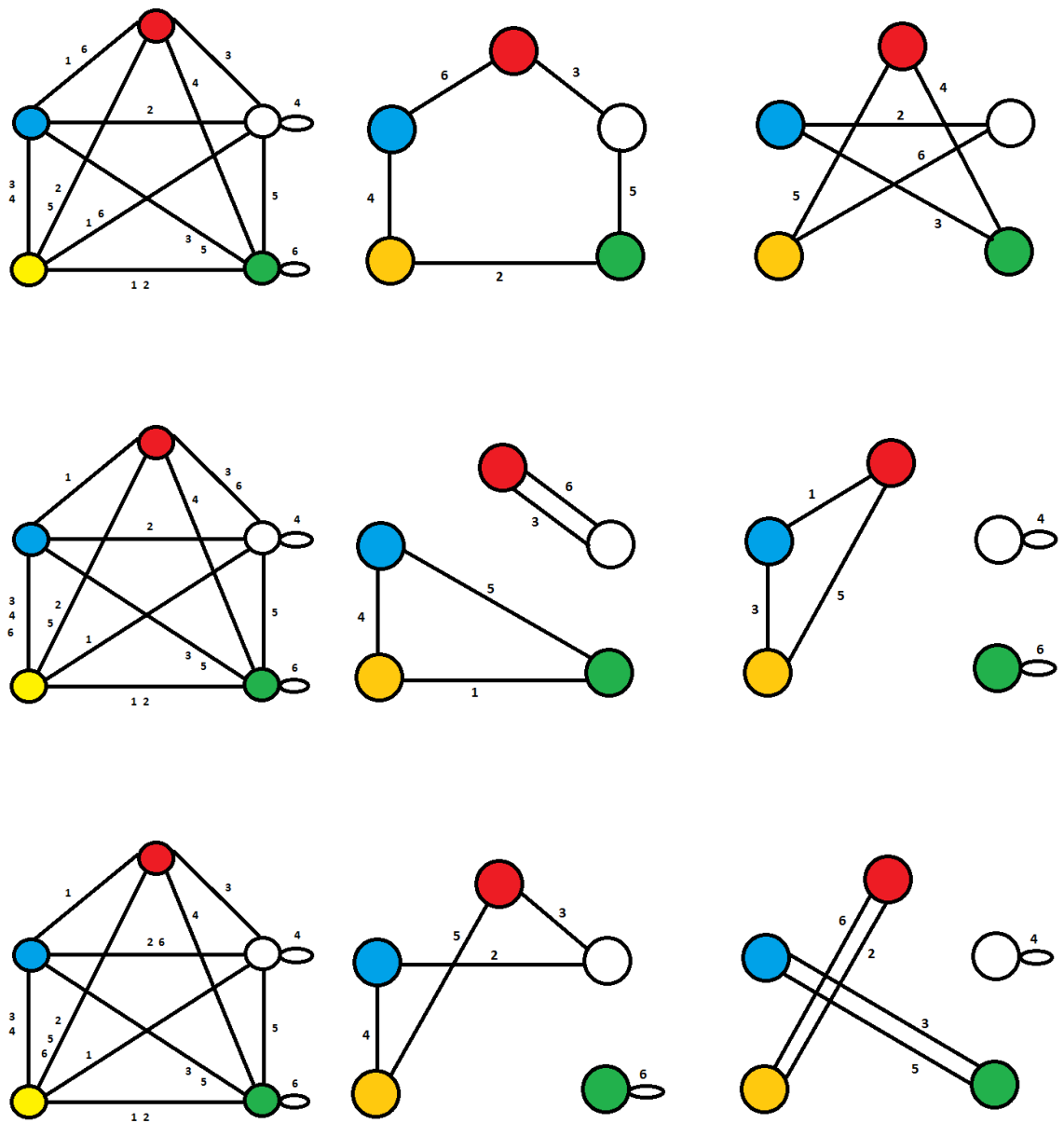


Figure 18. The multi-graphs and subgraph solutions of a self-loop on the green vertex.

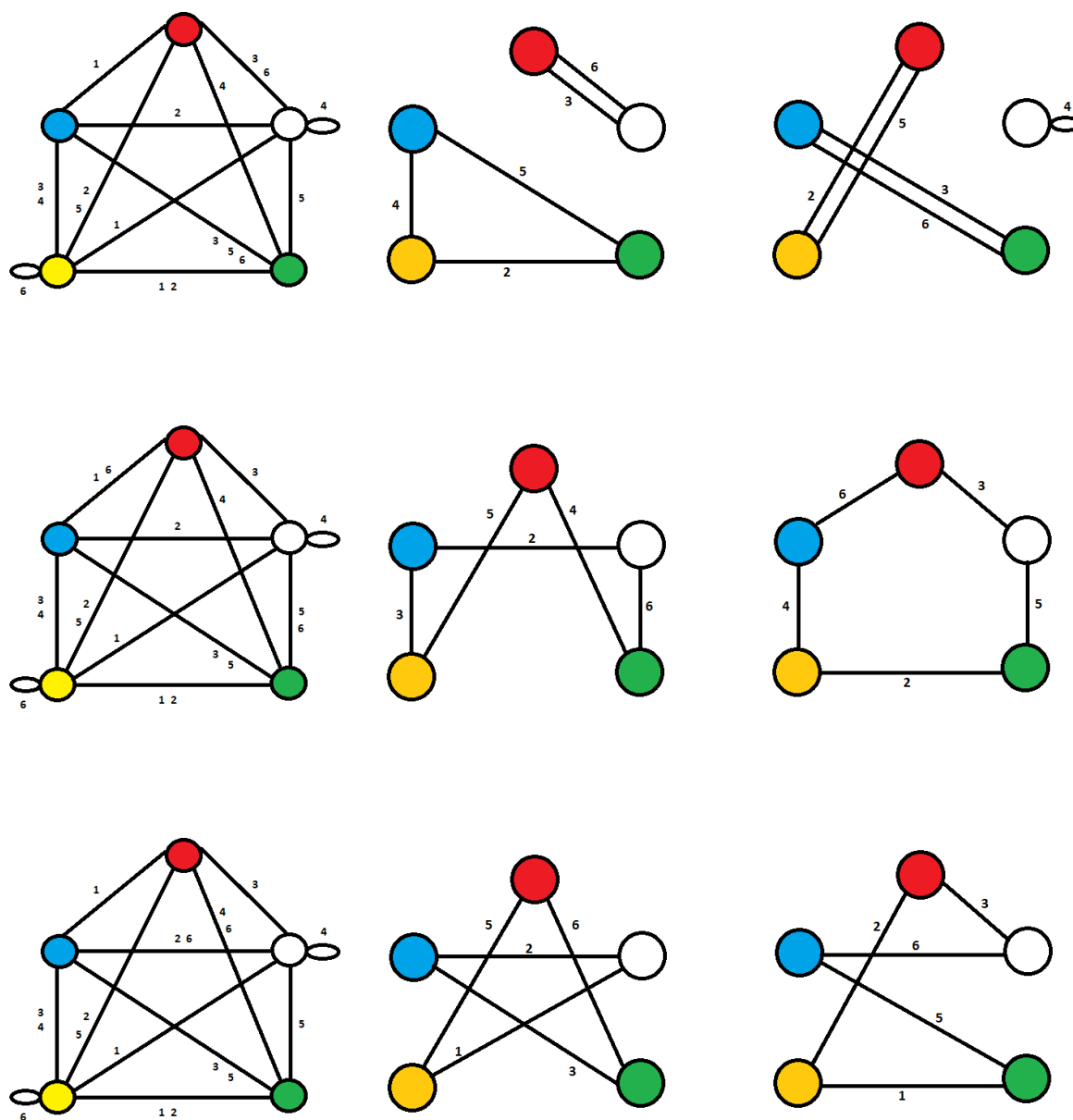


Figure 19. The multi-graphs and subgraph solutions of a self-loop on the yellow vertex.

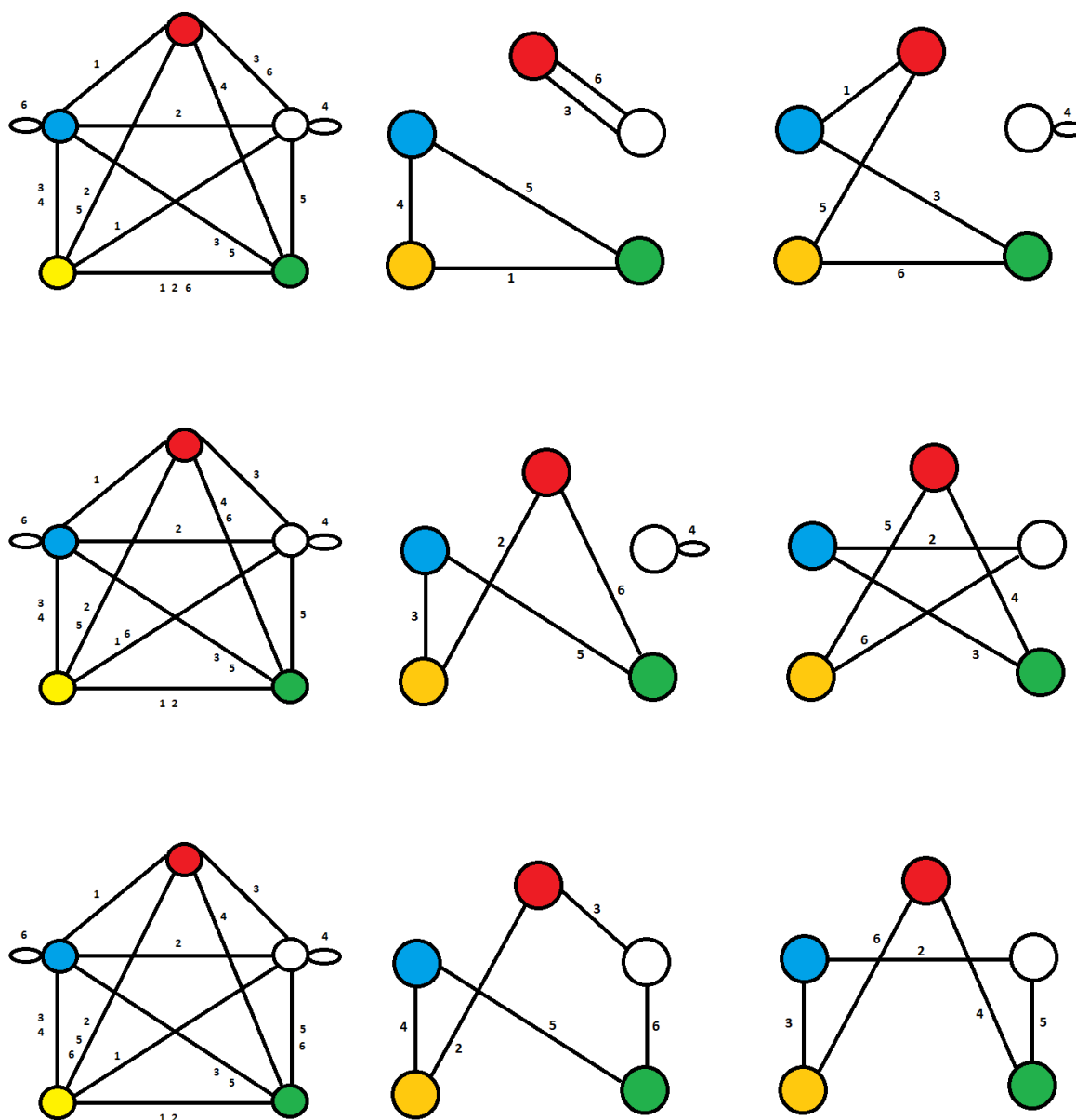


Figure 20. The multi-graphs and subgraph solutions of a self-loop on the blue vertex.

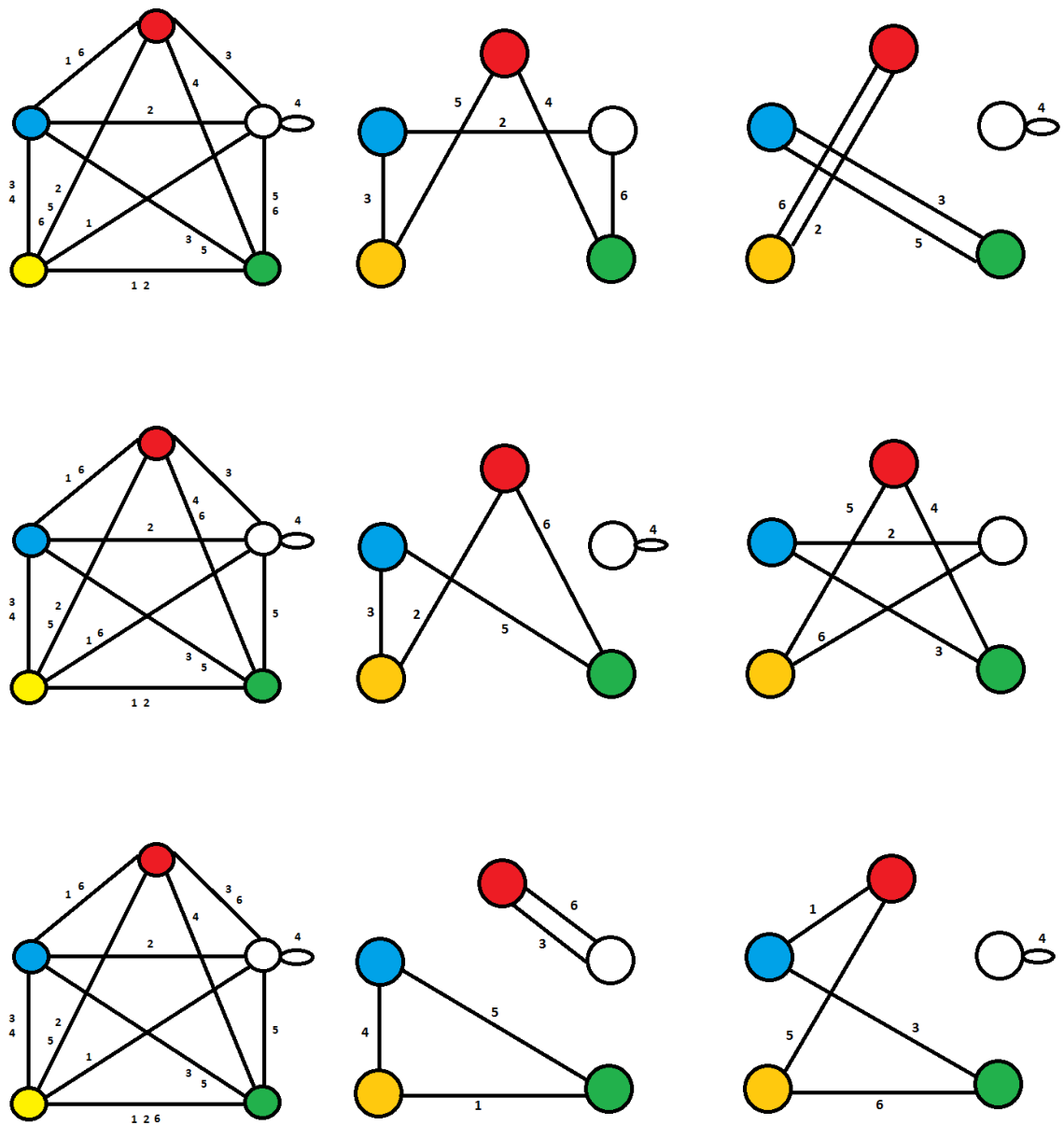


Figure 21.1. The multi-graphs and subgraph solutions of the double edge possibilities on the red vertex. The first three possibilities.

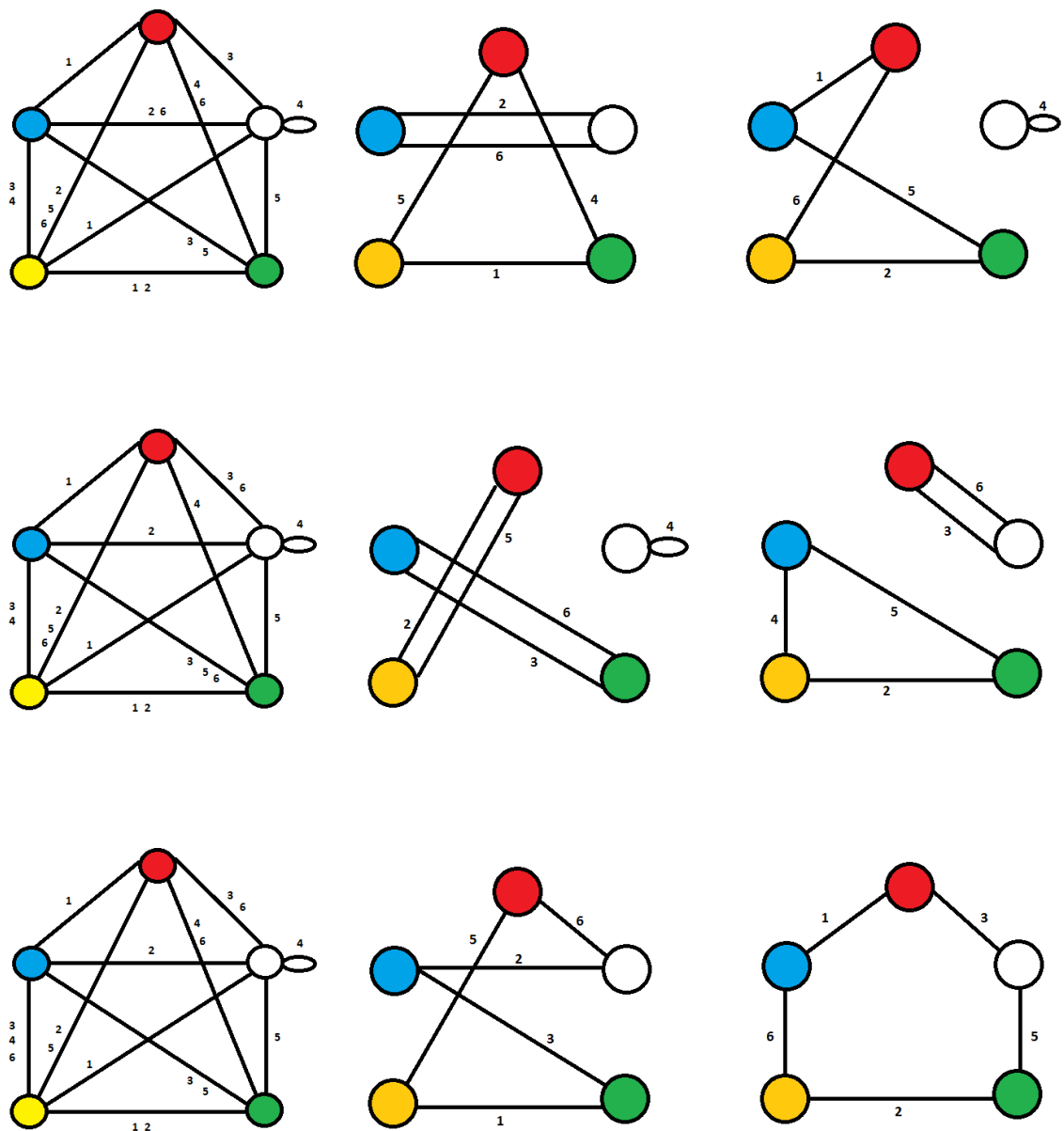


Figure 21.2 The multi-graphs and subgraph solutions of the double edge possibilities on the red vertex. The last three possibilities.

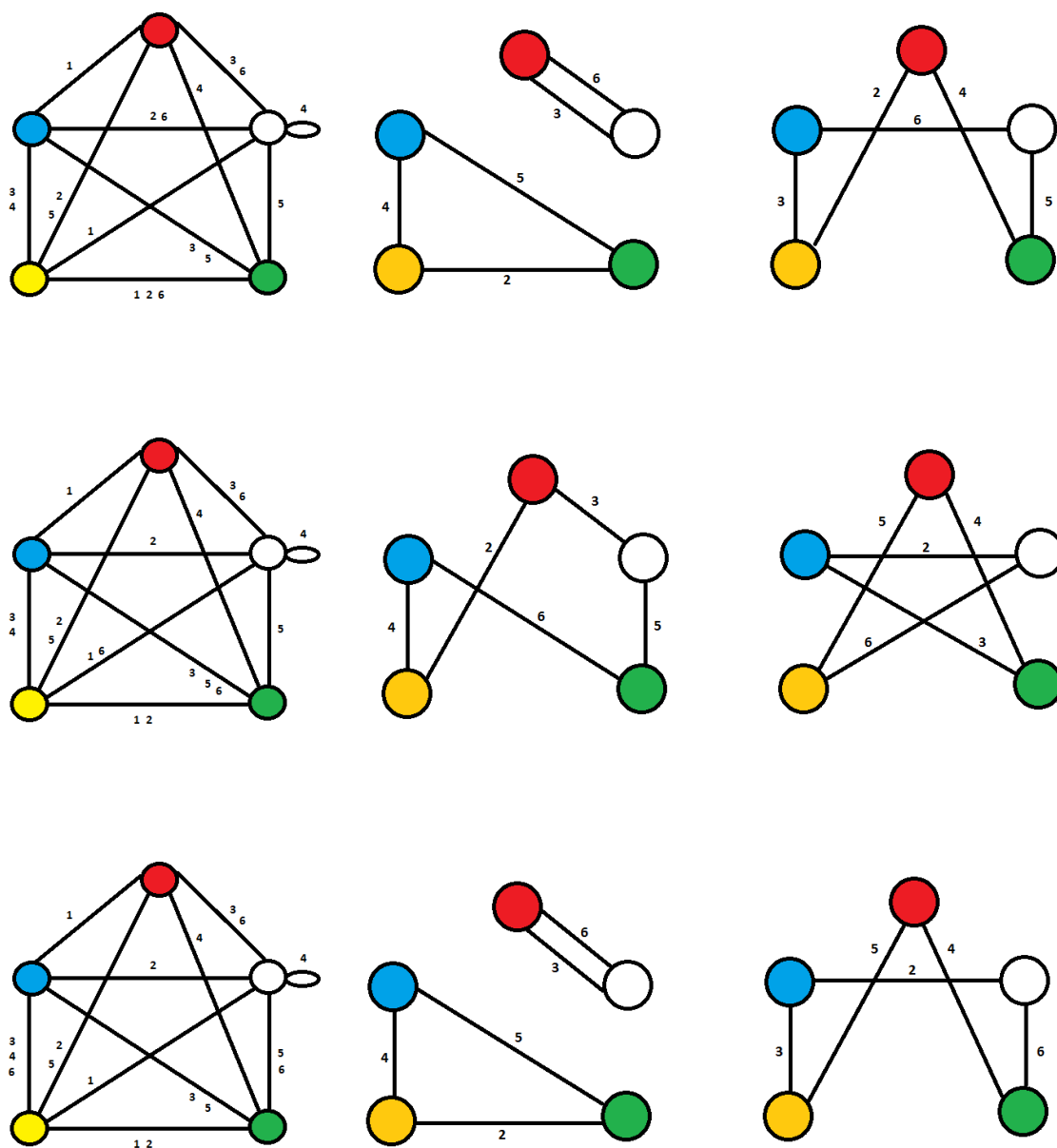


Figure 22.1 The multi-graphs and subgraph solutions of the double edge possibilities on the white vertex. The first three possibilities.

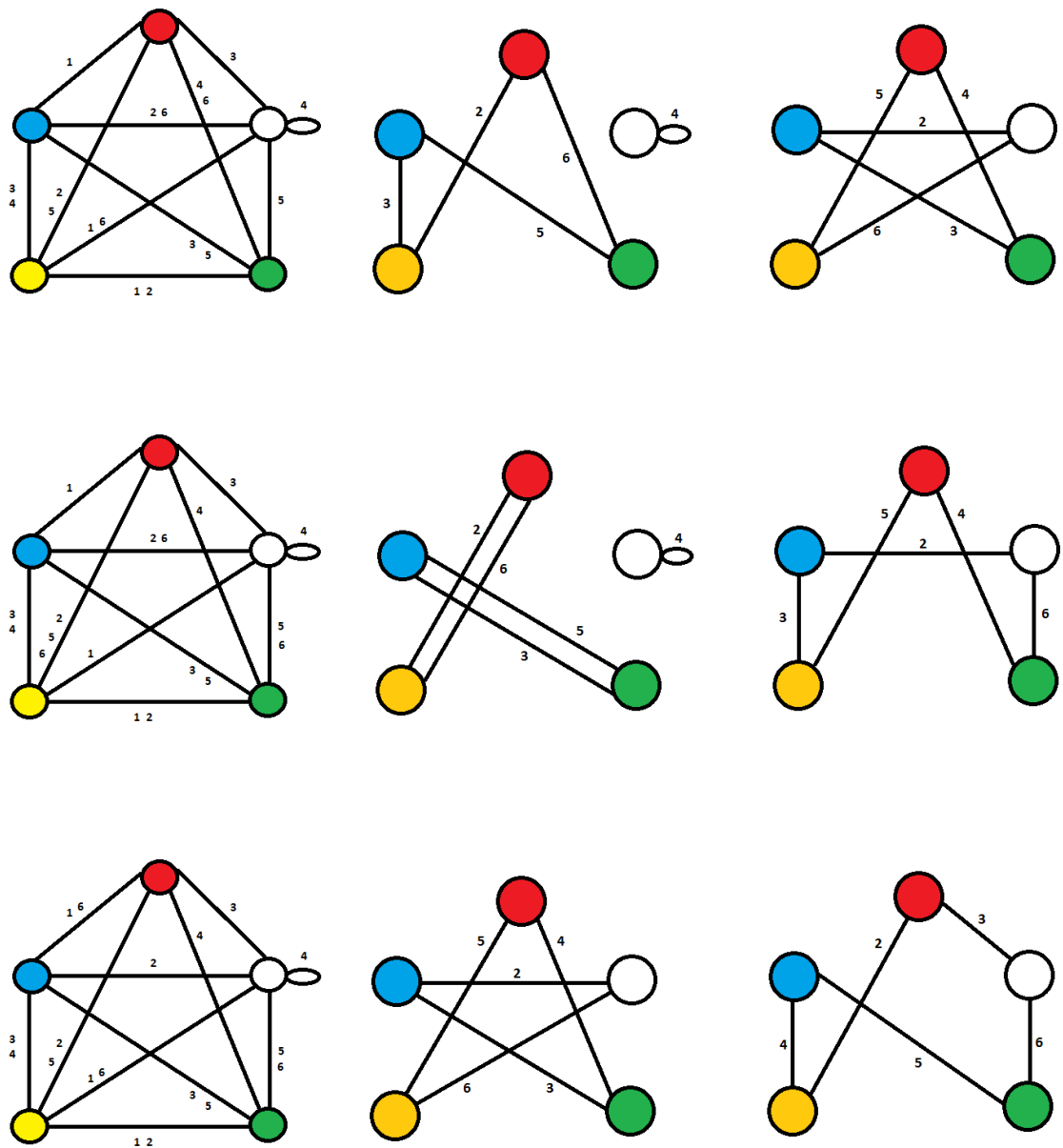


Figure 22.2. The multi-graphs and subgraph solutions of the double edge possibilities on the white vertex. The last three possibilities.

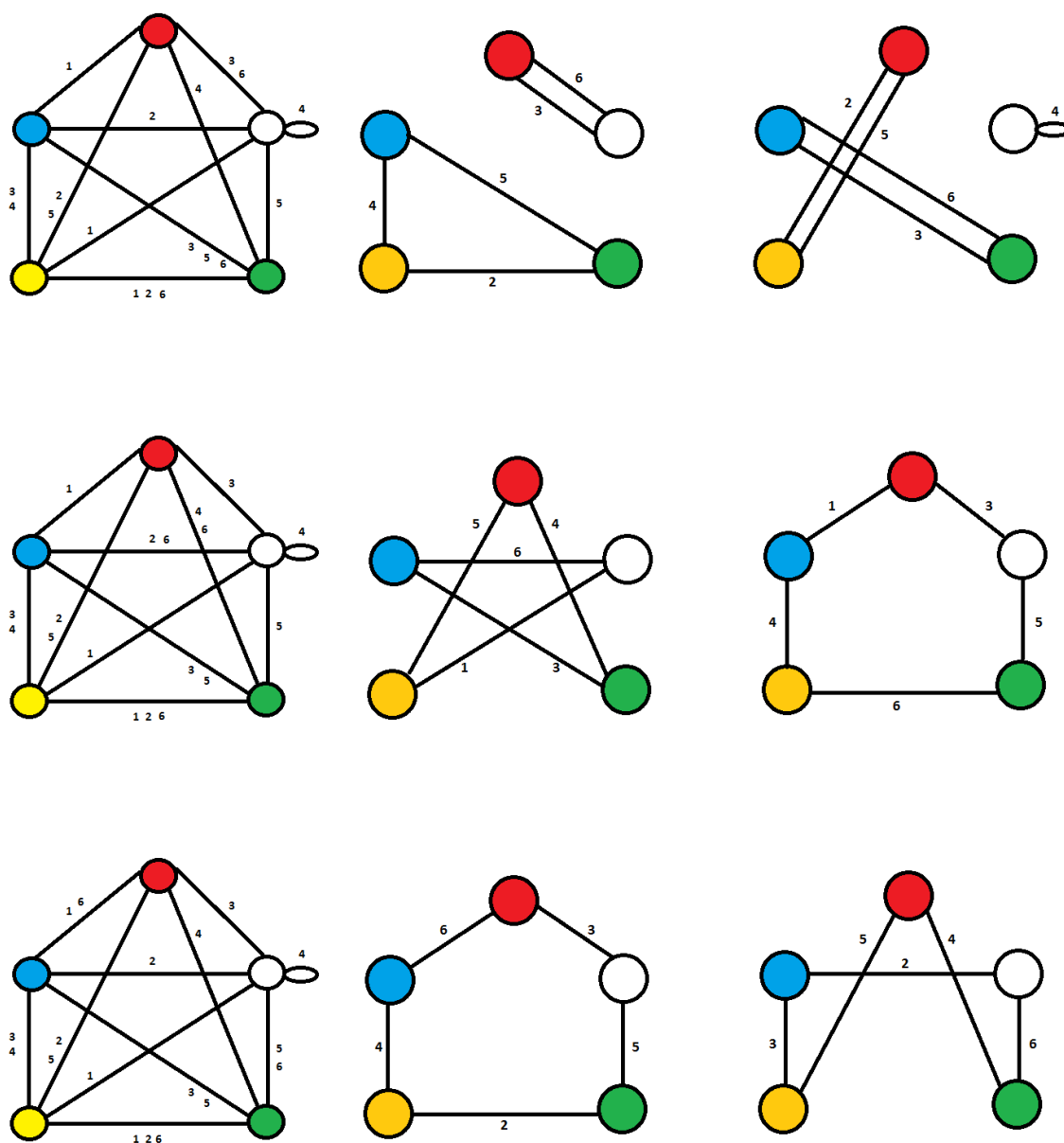


Figure 23.1. The multi-graphs and subgraph solutions of the double edge possibilities on the green vertex. The first three possibilities.

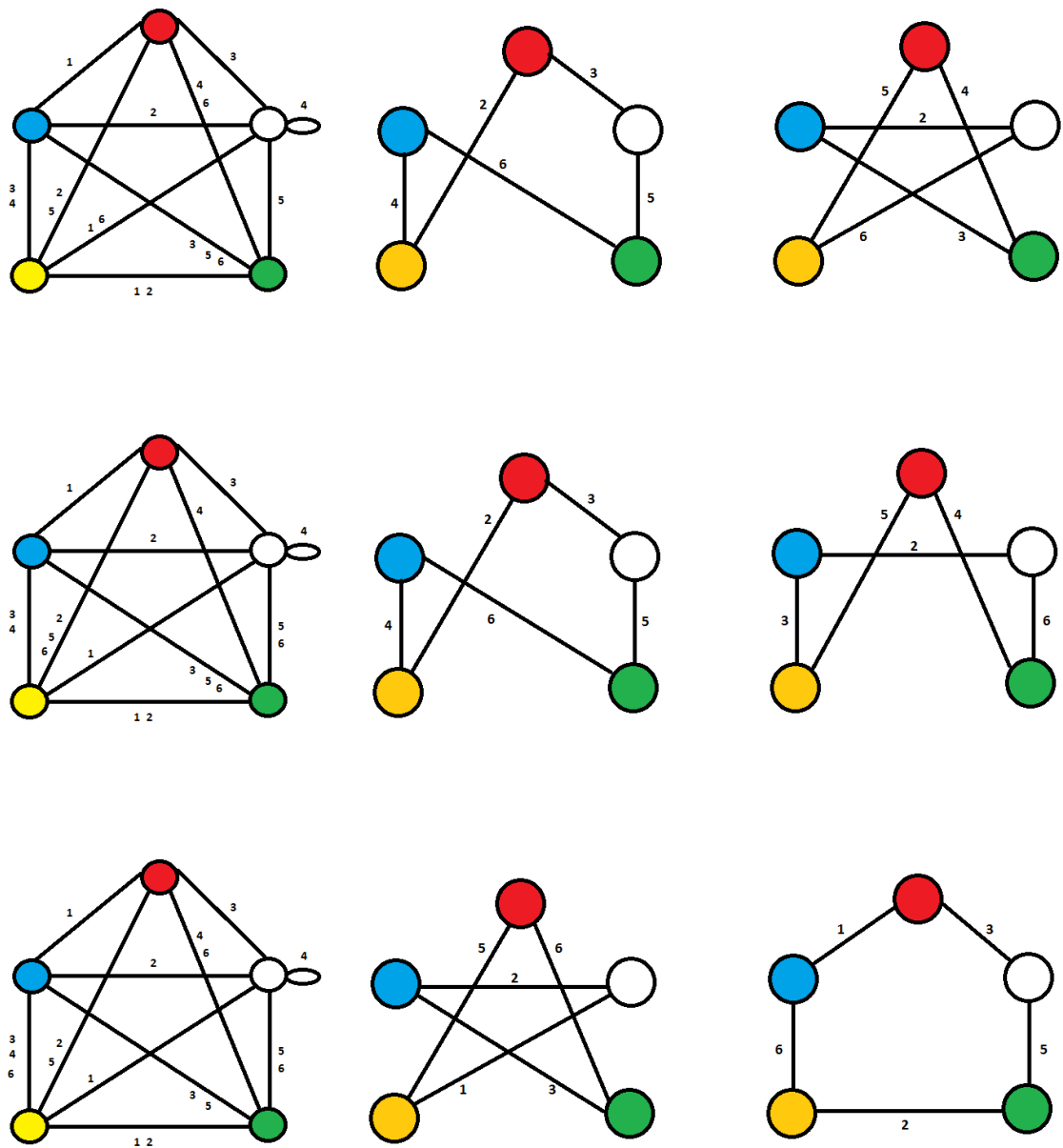


Figure 23.2. The multi-graphs and subgraph solutions of the double edge possibilities on the green vertex. The last three possibilities.

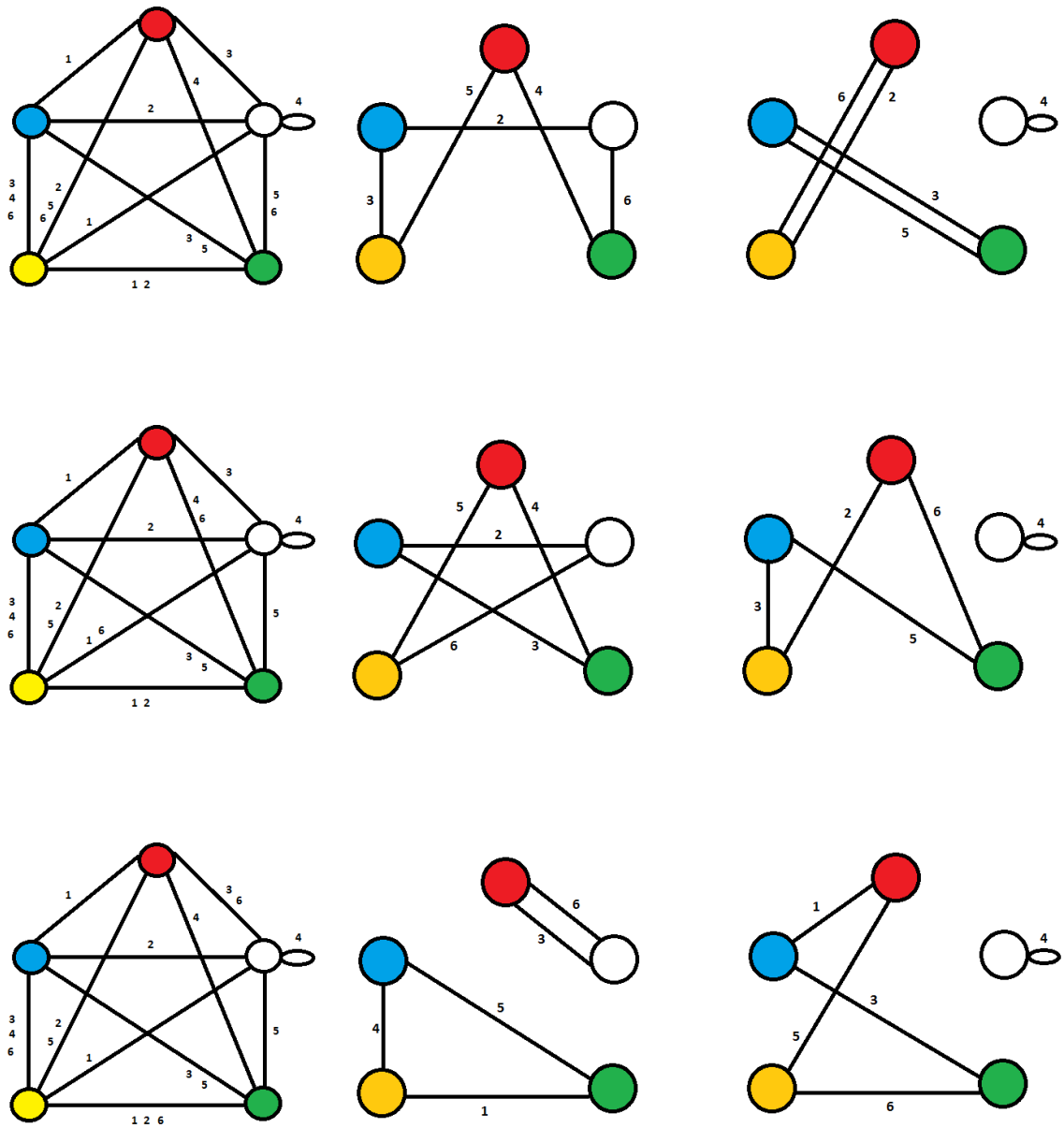


Figure 24.1. The multi-graphs and subgraph solutions of the double edge possibilities on the yellow vertex. The first three possibilities.

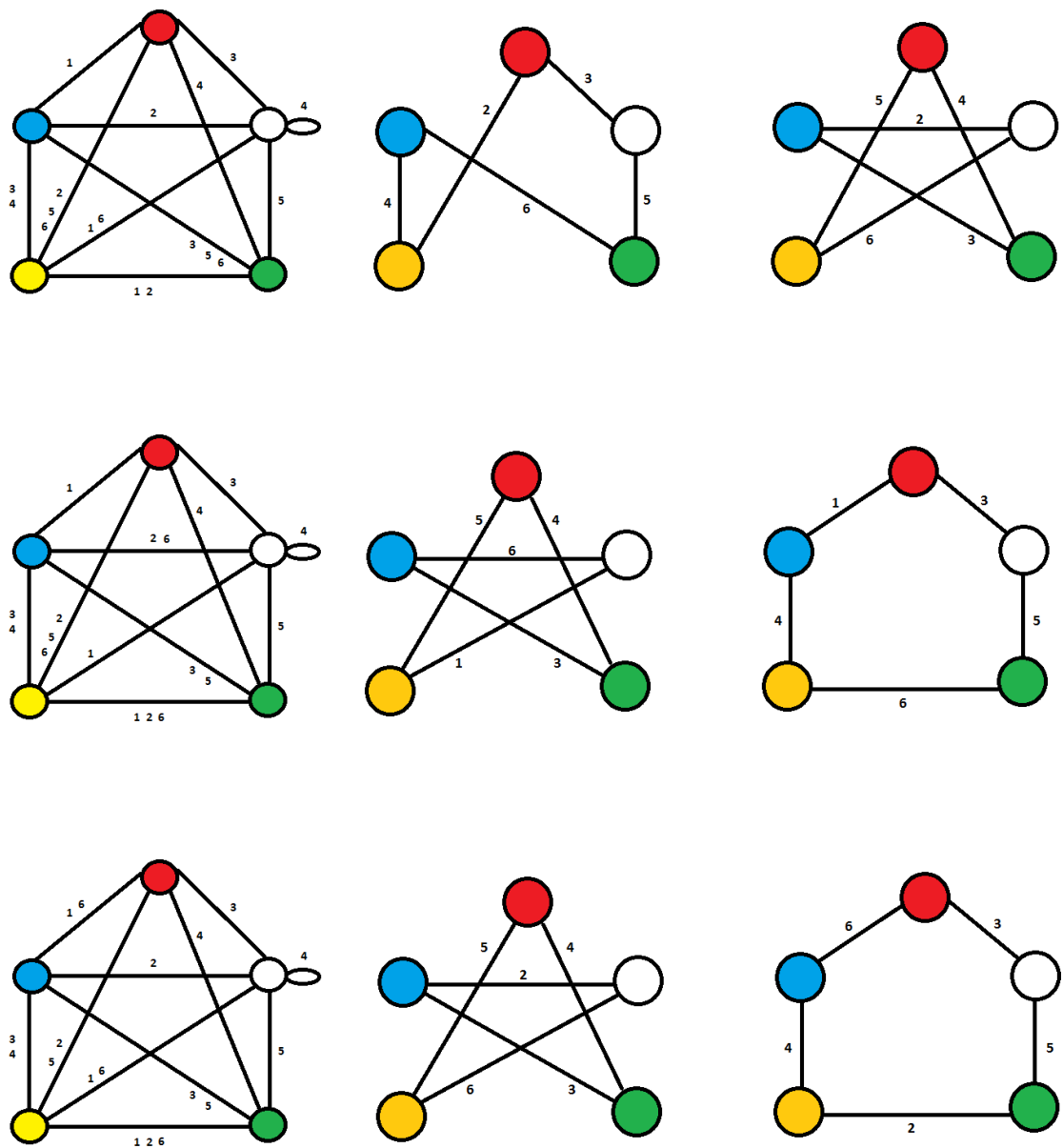


Figure 24.2. The multi-graphs and subgraph solutions of the double edge possibilities on the yellow vertex. The last three possibilities.

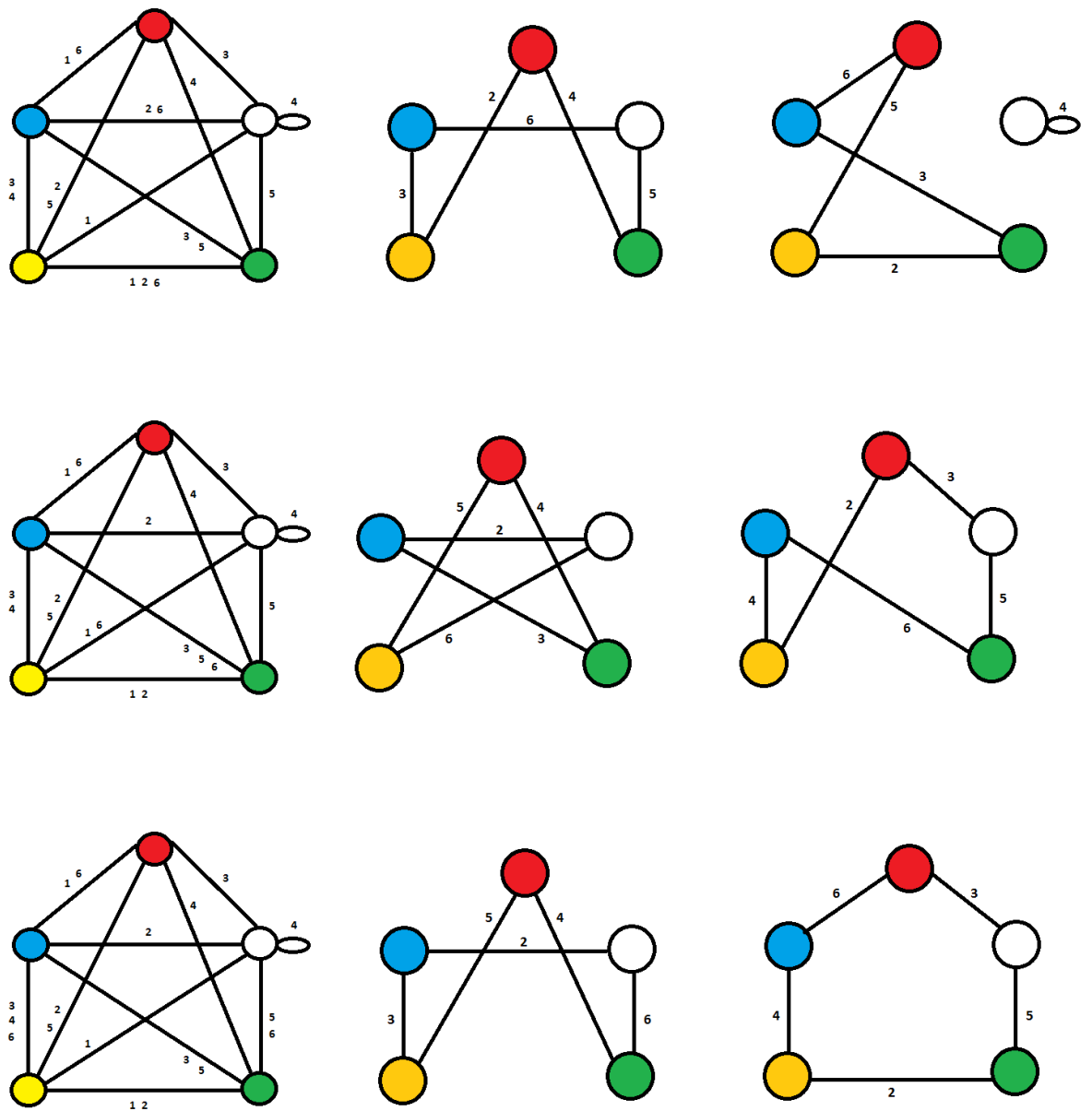


Figure 25.1. The multi-graphs and subgraph solutions of the double edge possibilities on the blue vertex. The first three possibilities.

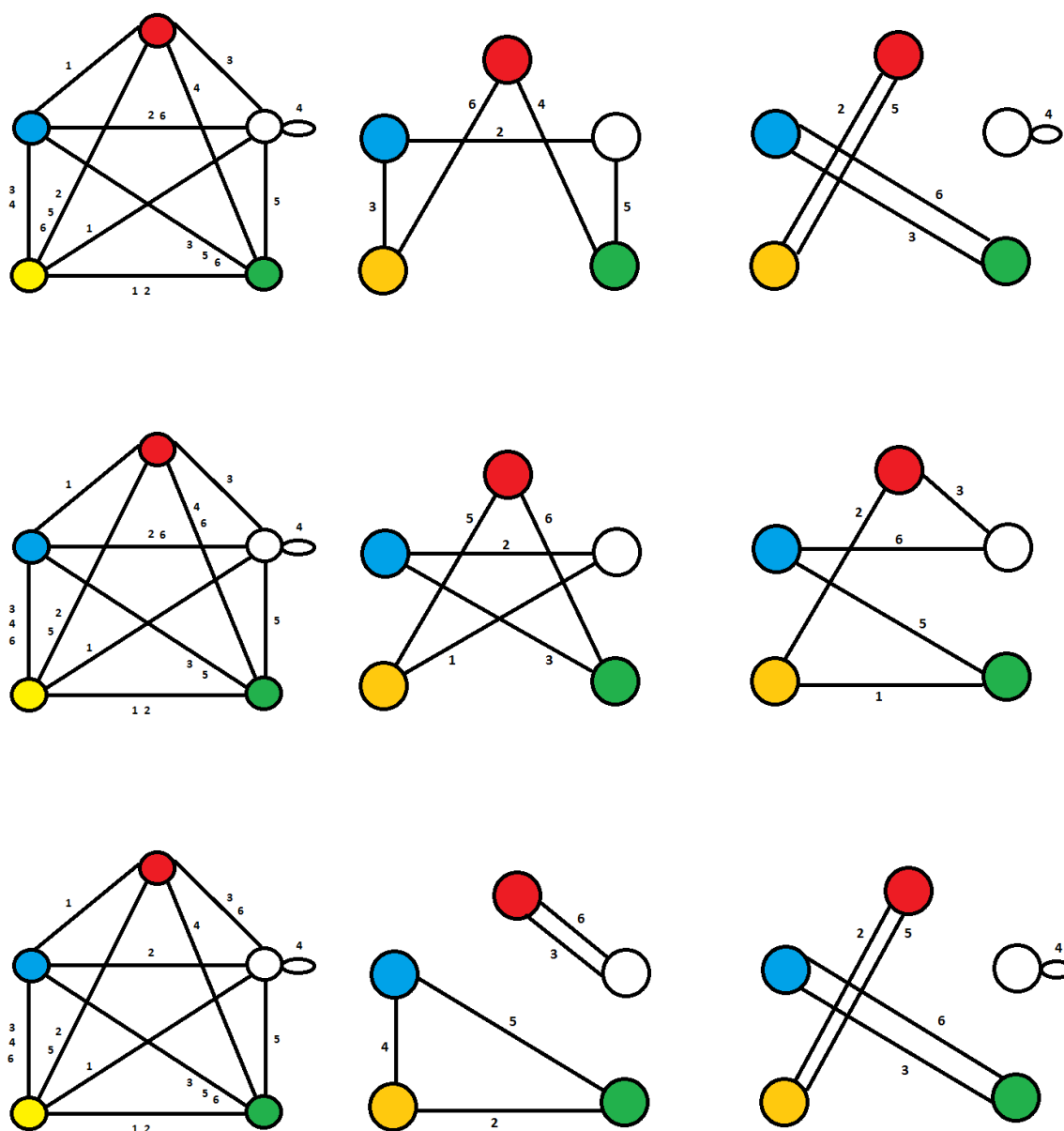


Figure 25.2. The multi-graphs and subgraph solutions of the double edge possibilities on the blue vertex. The last three possibilities.

### **Other Unique Solution Puzzles**

My program created fourteen non-isomorphic puzzles with a unique solution. Out of these fourteen puzzles, six of them have a no-solution cube associated with it. For the sake of space they will be placed in the appendix section. These puzzles were made by keeping four of the cubes and creating a new fifth one. By replacing the fifth cube, two new unique solutions were found. By replacing the fourth cube, five new unique solutions were found. Replacing the third cube leads to two new unique solutions. By replacing the second cube, four new unique solutions were found. By replacing the first cube, one new unique solution was found. The no-solution cubes for these puzzles were verified to yield no solution but their uniqueness was not tested.

### Chapter III

#### The Seven Choose Six Problem

When given six cubes with six colors a puzzle with a unique solution can be created. Unlike the five cube puzzle, these cubes do not necessarily have every color shown on every cube. The cubes are shown below in Figure 26 followed by their composite graph in Figure 27.

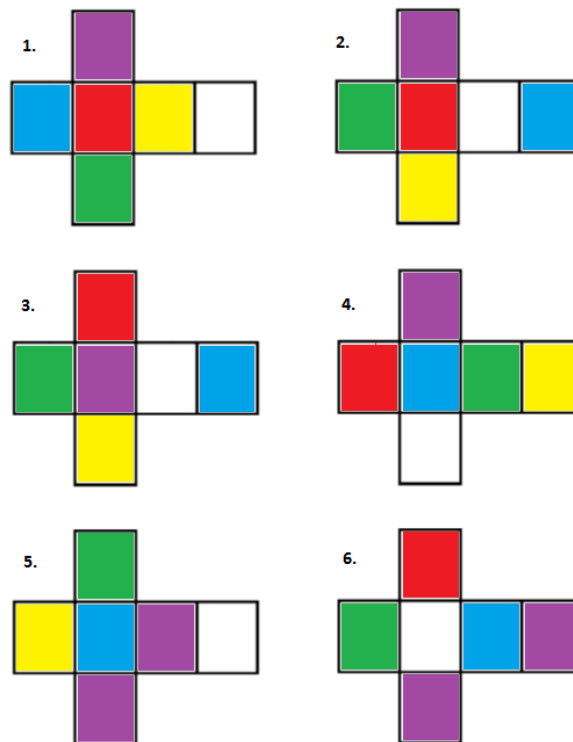


Figure 26. The set of cubes for the six cube problem.

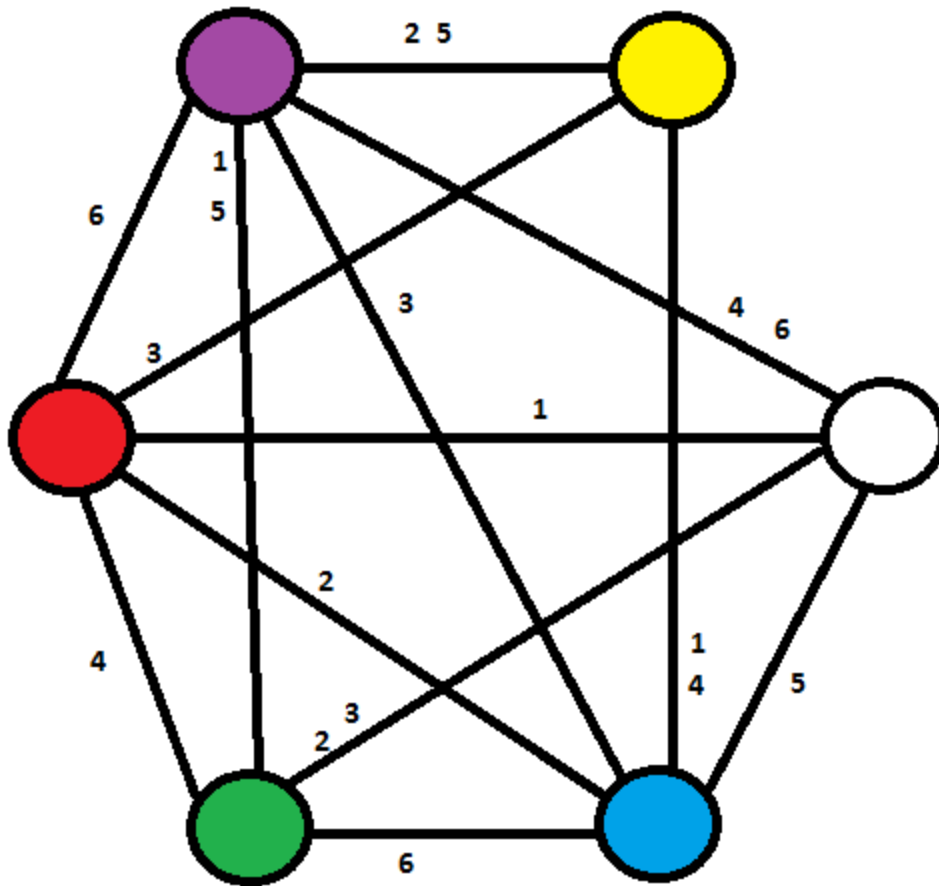


Figure 27. The multi-graph for the six cube problem.

As shown from the graph above, there is no color connection between yellow and white or green and yellow. This mapping is not isomorphic to any famous six cube puzzle that had their cubes available (Pattern, 2004; Storer, 2013).

The possible 2-regular spanning subgraphs of the six cube problem are shown in Figure 28.

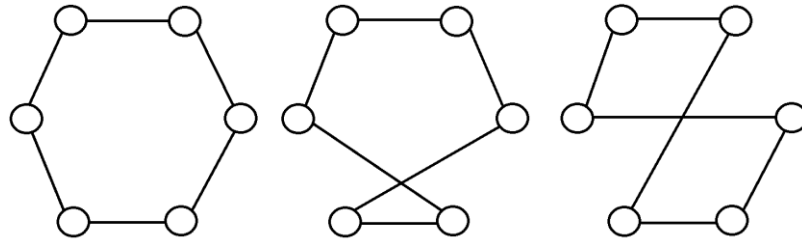


Figure 28.1. Three of the 2-regular subgraphs for the six cube problem.

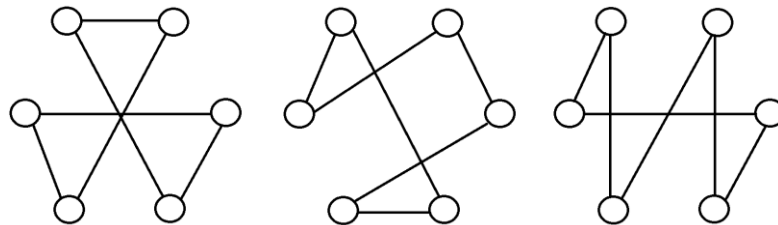


Figure 28.2. Three of the 2-regular subgraphs for the six cube problem.

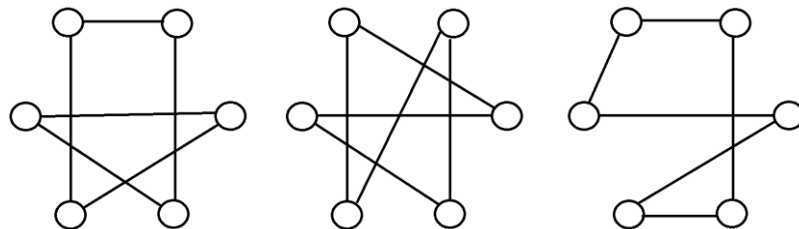


Figure 28.3. Three of the 2-regular subgraphs for the six cube problem.

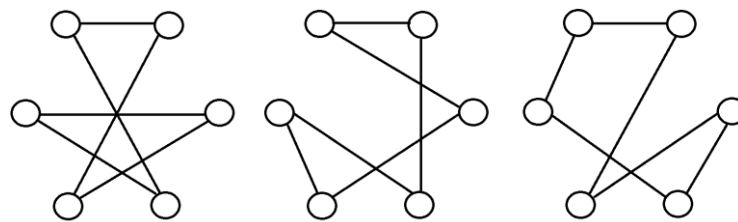


Figure 28.4. Three of the 2-regular subgraphs for the six cube problem.

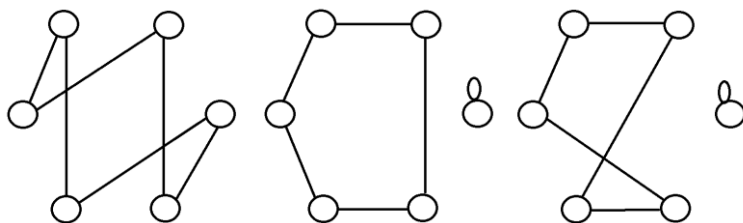


Figure 28.5. Three of the 2-regular subgraphs for the six cube problem.

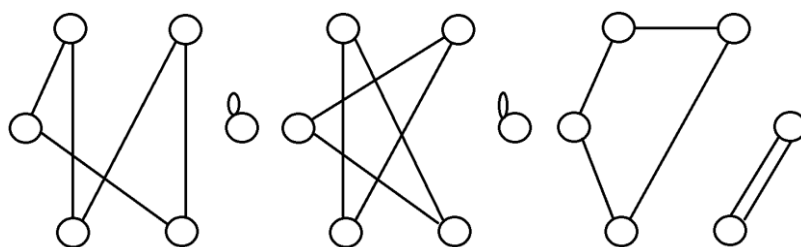


Figure 28.6. Three of the 2-regular subgraphs for the six cube problem.

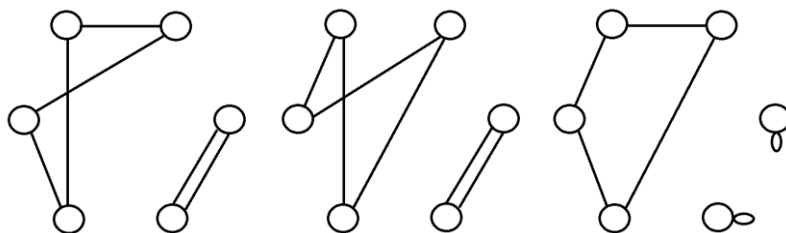


Figure 28.7. Three of the 2-regular subgraphs for the six cube problem.

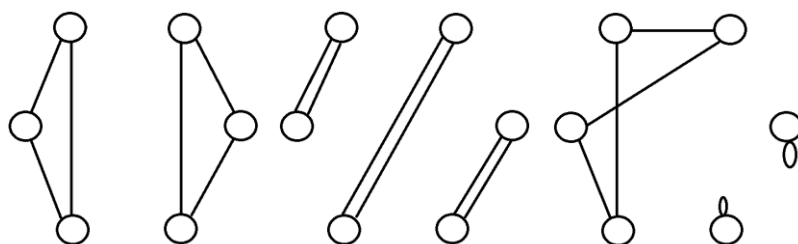


Figure 28.8. Three of the 2-regular subgraphs for the six cube problem.

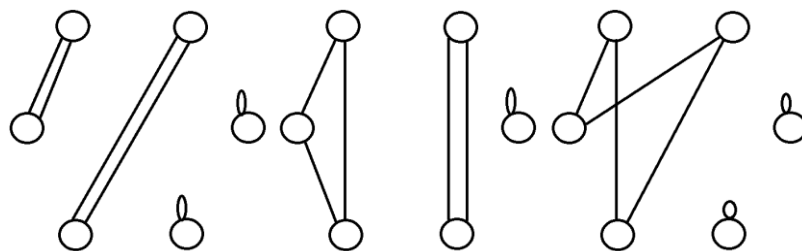


Figure 28.9. Three of the 2-regular subgraphs for the six cube problem.

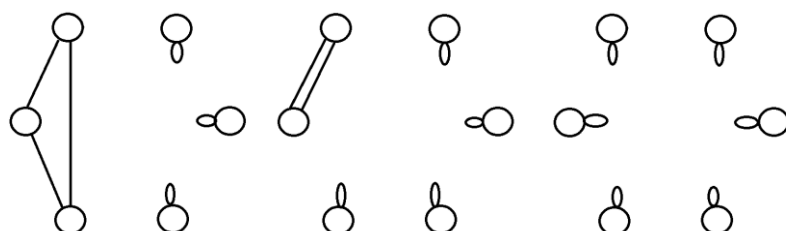


Figure 28.11. Three of the 2-regular subgraphs for the six cube problem.

After evaluating every subgraph and their rotations the following 2-regular spanning subgraphs are found.

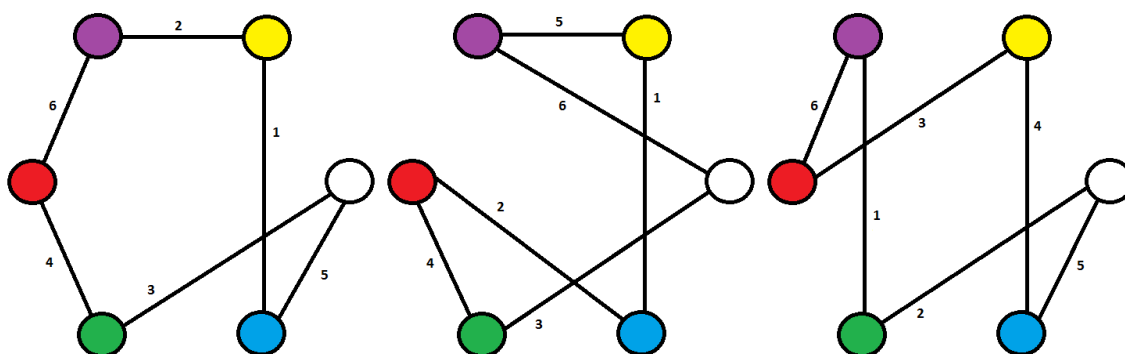
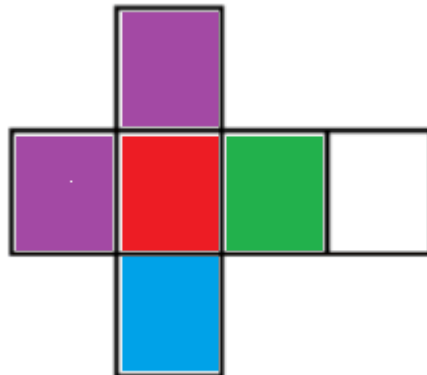


Figure 29. The subgraphs containing potential solutions to the six cube problem.

The first two subgraphs share edge 4. The first and third subgraphs share edge 6. The second and third subgraphs have no edge in common and are therefore the solution. Therefore this puzzle has a unique solution.

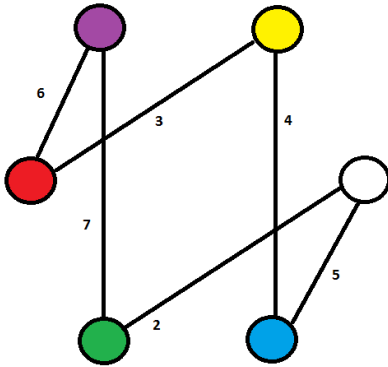
### The Seven Choose Six No Solution Cube

My program created a seventh cube such that when it replaces any of the six cubes from the previous problem it will make finding a solution impossible therefore creating the seven choose six problem.

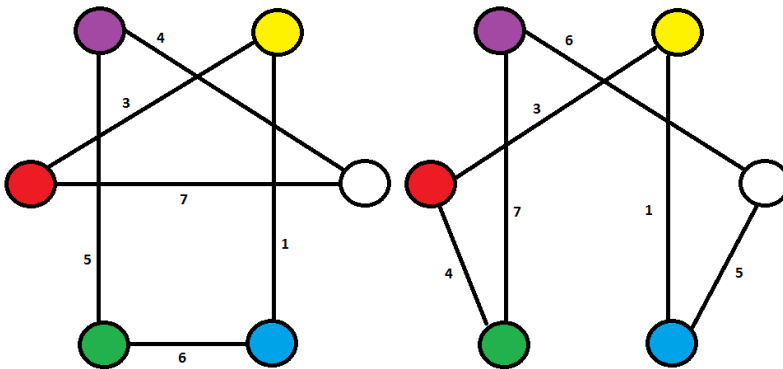


*Figure 30.* The no-solution cube for the seven choose six problem.

The subgraphs of potential solutions created by this cube are shown in Figures 31-35. Some sections only have one subgraph, making a solution impossible. It should be noted that replacing the third cube with the no solution cube yields no 2-regular spanning subgraphs to evaluate.



*Figure 31.* The subgraph created by replacing the first cube with the no-solution cube. There is no subgraph to pair it with so a solution is impossible.



*Figure 32.* The subgraphs created by replacing the second cube with the no solution cube. These subgraphs have edge 1 in common so they do not represent a solution.

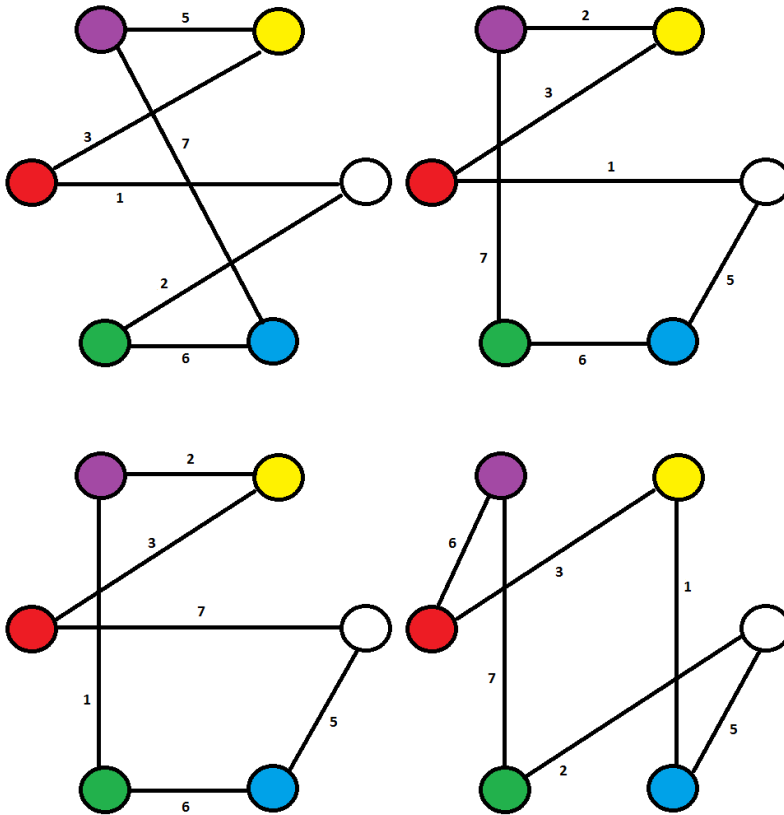


Figure 33. The subgraphs found by replacing the fourth cube with the no-solution cube. It is left to the reader to see why there is no solution from these subgraphs.

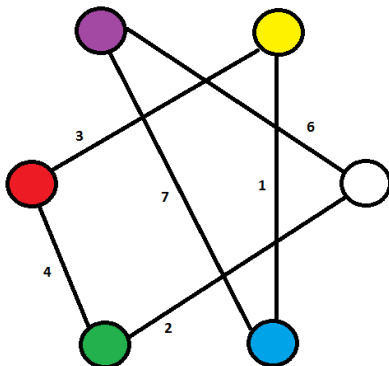
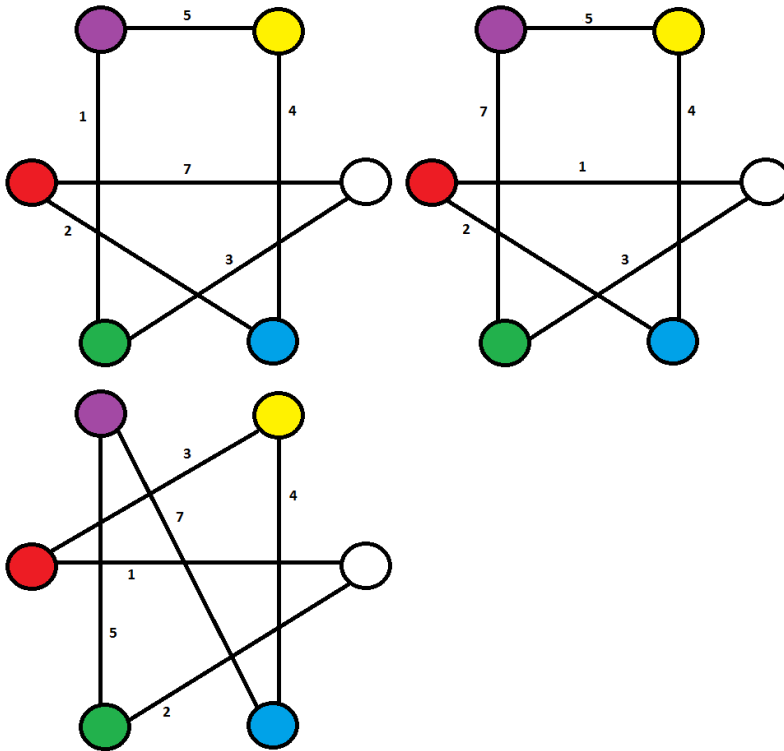


Figure 34. The subgraph found by replacing the fifth cube with the no-solution cube. Since there is no 2-regular spanning subgraph to compare it with a solution is impossible.



*Figure 35.* The subgraphs found by replacing the sixth cube with the no-solution cube. All of these subgraphs share edge 4 so therefore a solution is impossible.

Overall, my program was able to generate six six cube problems with unique solutions.

However, only two of these puzzles had a no-solution cube. This does not mean that those cubes do not exist for the other puzzles. These puzzles will be listed in the appendix.

## APPENDIX A

## Other Isomorphic Five Cube Puzzles With Unique Solutions

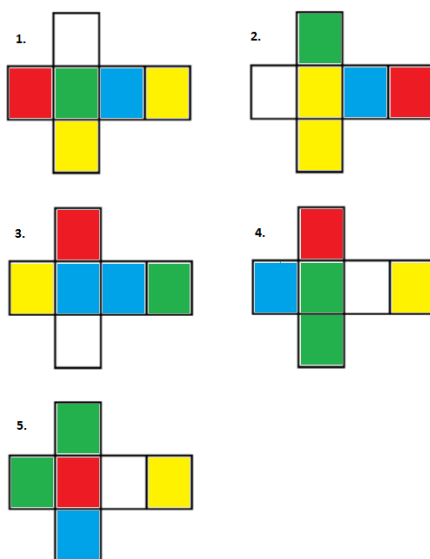


Figure A-1. A new five cube puzzle without a no-solution cube.

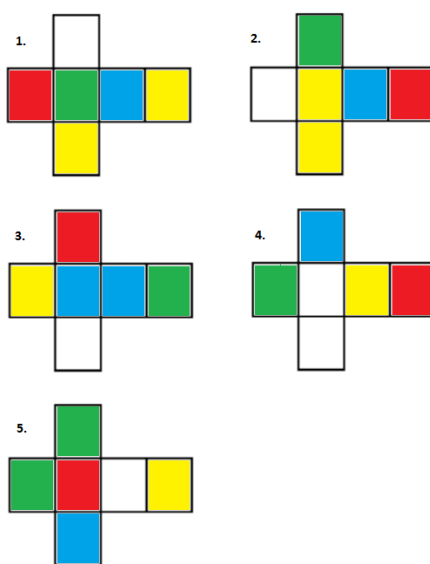


Figure A-2. A new five cube puzzle without a no-solution cube.

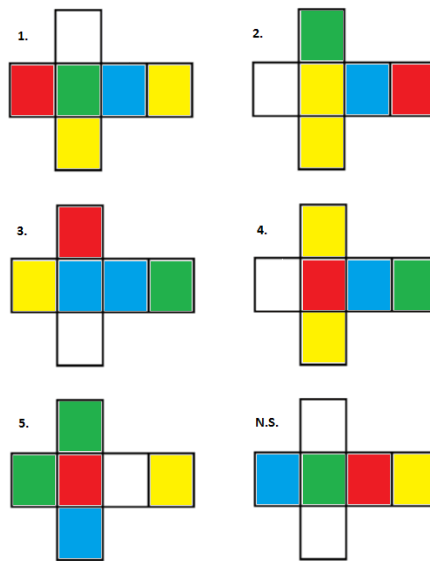


Figure A-3. A new five cube puzzle with a no-solution cube.

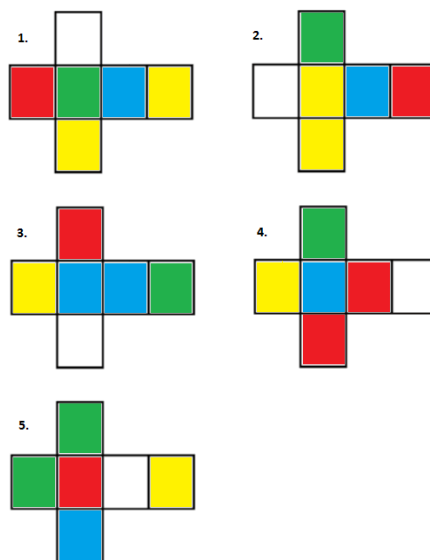


Figure A-4. A new five cube puzzle without a no-solution cube.

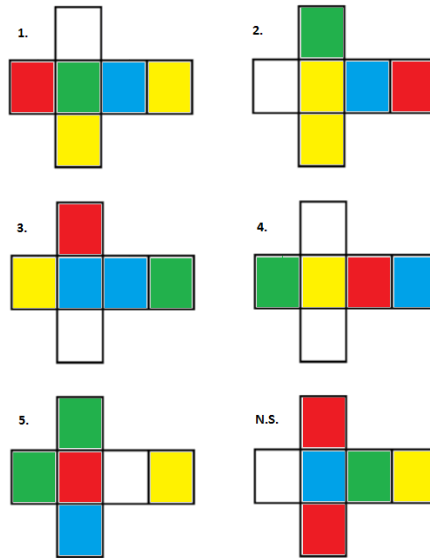


Figure A-5. A new five cube puzzle with a no-solution cube.

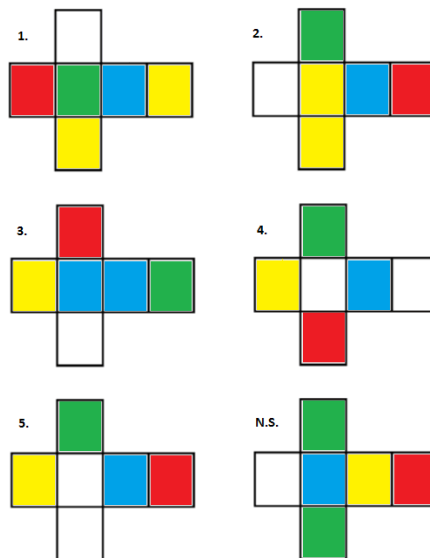


Figure A-6. A new five cube puzzle with a no-solution cube.

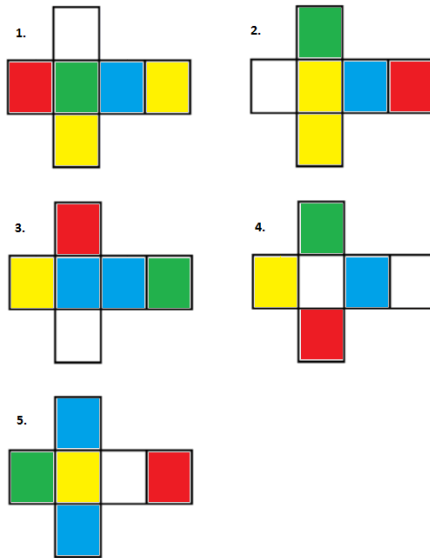


Figure A-7. A new five cube puzzle without a no-solution cube.

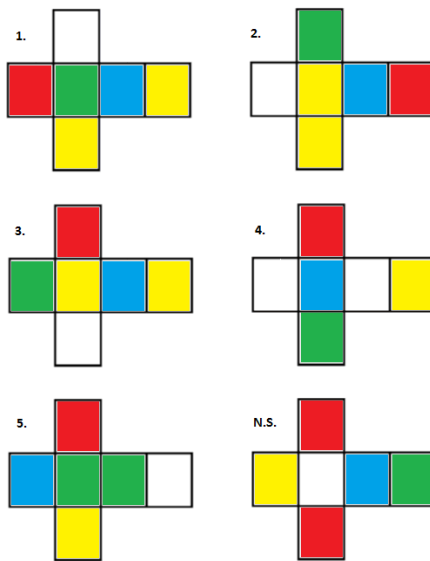


Figure A-8. A new five cube puzzle with a no-solution cube.

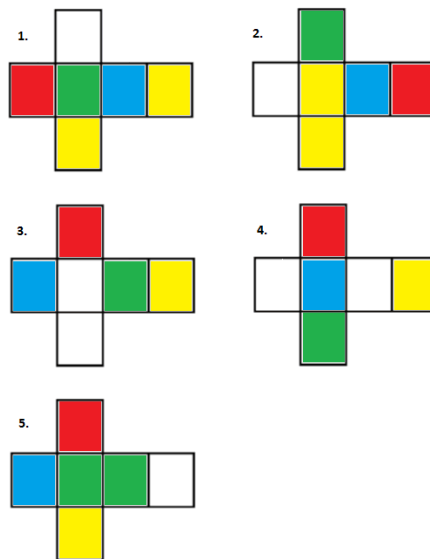


Figure A-9. A new five cube puzzle without a no-solution cube.

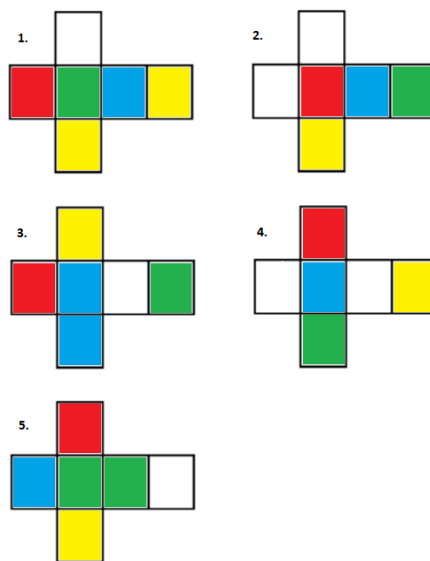


Figure A-10. A new five cube puzzle without a no-solution cube.

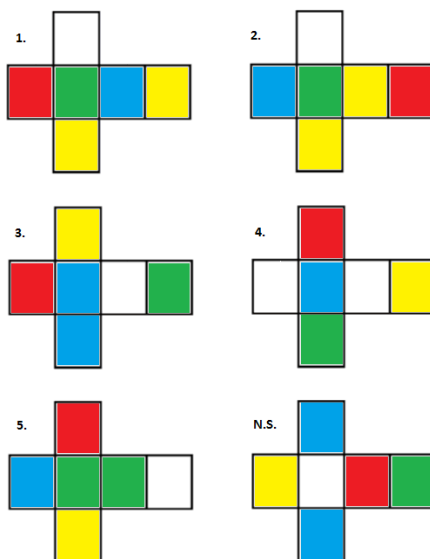


Figure A-11. A new five cube puzzle with a no-solution cube.

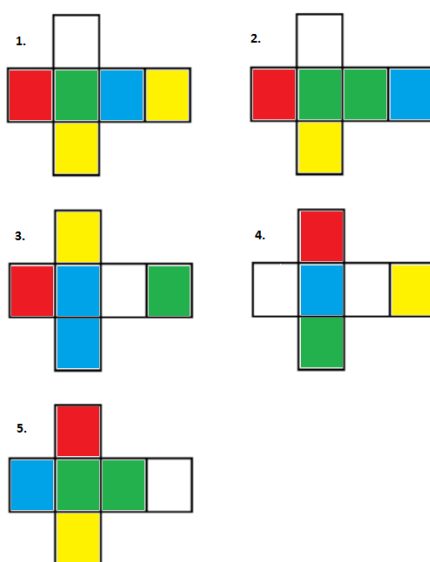


Figure A-12. A new five cube puzzle without a no-solution cube.

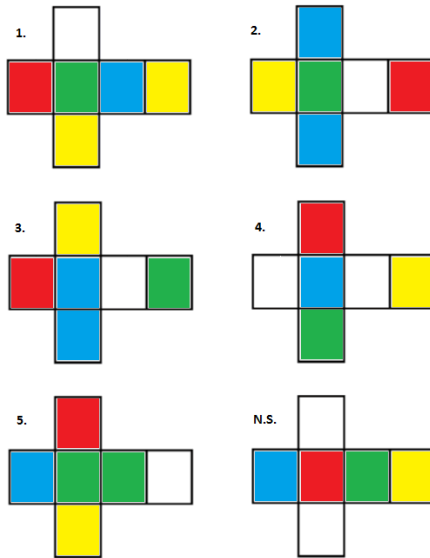


Figure A-13. A new five cube puzzle with a no-solution cube.

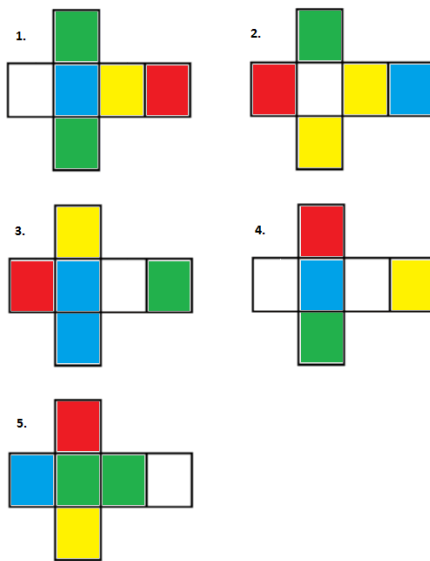


Figure A-14. A new five cube puzzle without a no-solution cube.

APPENDIX B

Alternative Isomorphic Six Cube Puzzles With Unique Solutions

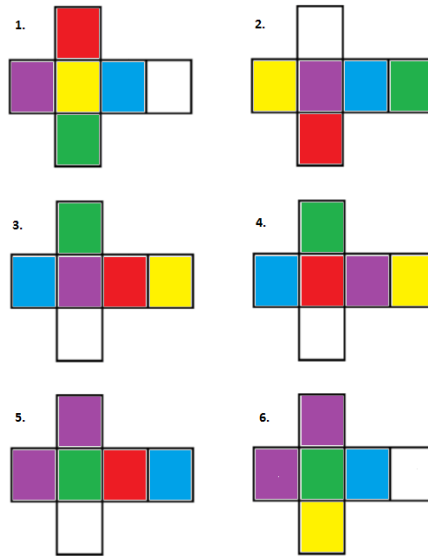


Figure B-1. A new six cube puzzle without a no-solution cube.

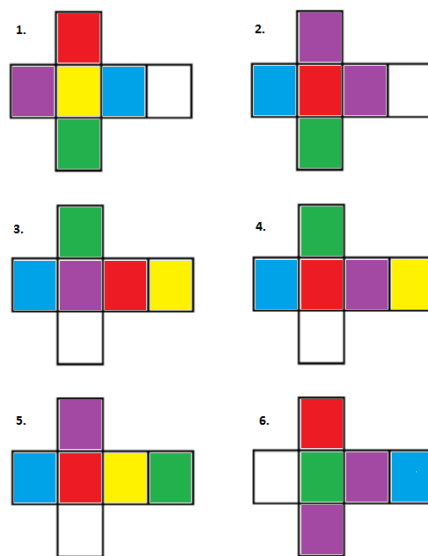


Figure B-2. A new six cube puzzle without a no-solution cube.

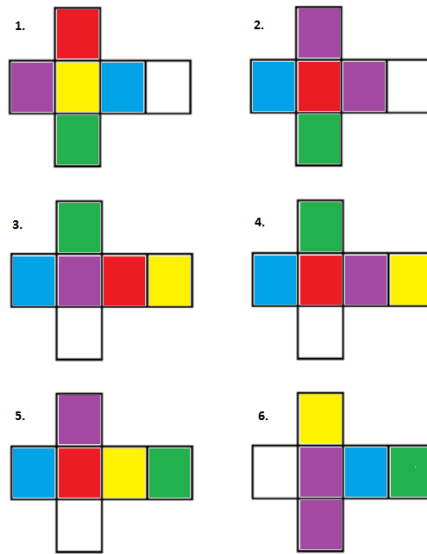


Figure B-3. A new six cube puzzle without a no-solution cube.

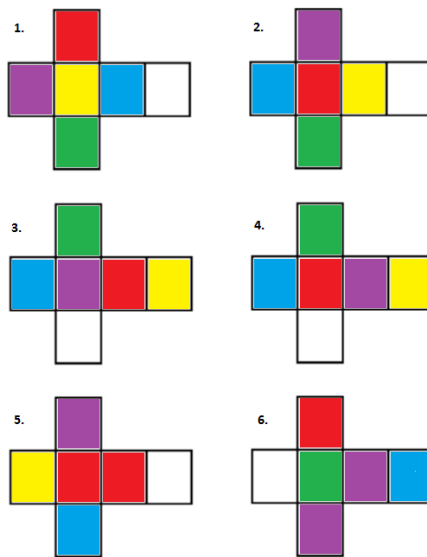


Figure B-4. A new six cube puzzle without a no-solution cube.

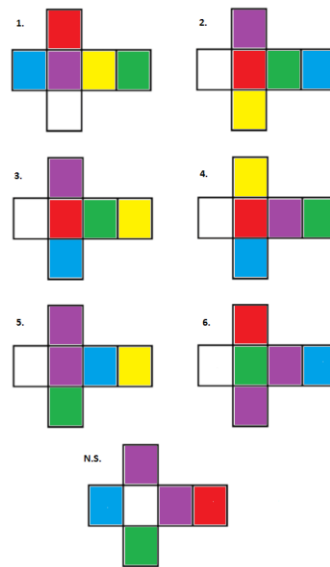


Figure B-5. A new six cube puzzle with a no-solution cube.

## References

Chartrand, G., & Zhang, P. (2005). *Introduction to graph theory*. Boston: McGraw-Hill Higher Education.

Hinder, M. (2014). Four, Five, and Six Cube Programs. Retrieved from <https://sites.google.com/site/fourfiveandsixcubeprograms/>

Koenecke, N. & Winters, S.J. (n.d.). *A five choose four Instant Insanity puzzle with a unique solution*. Manuscript submitted for publication.

Stegmann, R. (2013, November 15). Rob's Puzzle Page: Pattern Puzzles. Retrieved from <http://home.comcast.net/~stegmann/pattern.htm>

Schossow, F. A. (1900). *Patent Identifier No. 646,463*. Washington, DC: US Patent and Trademark Office.

Storer, J. A. (2014, March 20). Mechanical Puzzles. Retrieved from <http://www.cs.brandeis.edu/~storer/JimPuzzles/indexBOOK.html>