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**Quality Quandaries - Six Sigma,  
Process Drift, Capability Indices,  
and Feedback Adjustment**

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## **Quality Quandaries - Six Sigma, Process Drift, Capability Indices, and Feedback Adjustment**

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### ***ABSTRACT***

The Six Sigma specification makes an allowance of 1.5 standard deviations for process drift. Simple ways in which a major part of such drift can be removed are given. These employ feedback adjustment methods specifically designed for SPC applications. Key Words: Six Sigma, Process Drift, Capability Index, Feedback Adjustment

# Quality Quandaries\* - Six Sigma, Process Drift, Capability Indices, and Feedback Adjustment\*\*

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## Abstract

*The Six Sigma specification makes an allowance of 1.5 standard deviations for process drift. Simple ways in which a major part of such drift can be removed are given. These employ feedback adjustment methods specifically designed for SPC applications. Key Words: Six Sigma, Process Drift, Capability Index, Feedback Adjustment.*

## Six Sigma

The Six Sigma initiative (see e.g. Smith 1992; Harry 1994; Hoerl 1998) is seen by many as the blueprint for quality improvement for today and for the future.

That all humankind and hence all employees at every level have the natural ability to be creative has long been understood. But it has taken some time for the enormous potential of this fact to be fully appreciated. At last, in some of the most important companies, management has come to understand that their chief responsibility is to foster such efforts. They have done this, in part, by:

- a) making it absolutely clear that quality improvement is part of each person's everyday job,
- b) providing appropriate training at all levels in the organization,
- c) making quality improvement a competitive sport using "Managerial Champions", "Black Belts" and so on.

It is not surprising to learn that these principles rigorously applied in such companies as Motorola, G.E., Texas Instruments, Polaroid and Allied Signal have produced impressive results—for example, the 1997 annual report of Allied Signal attributes a savings of about 1.5 billion dollars to their Six Sigma initiative.

## Process Drift

This article is about how such efforts might be further improved. The Six Sigma initiative uses the many tools for quality improvement to ensure that all but a tiny fraction of manufactured articles are within specification limits. To achieve this, it is required that the process is so well controlled that each of these limits is six standard deviations away from the target. This corresponds to the use of a capability index (CP) of 2 or equivalently a spec/sigma ratio of 12 (Box and Luceño 1997)<sup>1</sup>. The stated goal is to produce no more than 3.4 defects per million. On the assumption that the process distribution is Normal and has a *fixed mean value that is on target*, this would require a spec/sigma ratio much less than 12. However, the authors of the Six Sigma concept wisely assume that the process does *not* have a fixed mean value but undergoes drift — specifically that the local mean may drift on either side of the target by about 1.5 standard deviations. Conventional wisdom would say that such process drift must occur from special causes and that these ought to be tracked down and eliminated. But in practice such drift, although detectable, may be impossible to assign or to eliminate economically.

It is refreshing to see, at last, this acknowledgment that, *even when best efforts are made* using standard quality control methods, the process mean can be expected to drift. In the

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<sup>1</sup> Later referenced in this article as B.L.

evolution of ideas about quality this new provision for process drift comes much closer to reality. It is a major departure from what, for the last fifty years, many have believed was a fundamental concept of statistical quality control.

The ideas of Shewhart and Deming concerning process monitoring are based on the approximation that in the absence of “special causes” the process is in a state of control with fixed mean. However, as has been pointed out elsewhere (see e.g. B.L.), the second law of thermodynamics ensures that no process could ever be in a state of control about a fixed mean. Furthermore, the long term instability of processes — after standard techniques of quality control have been applied — has been verified by extensive studies of process data (see, for example, Alwan and Roberts, 1988). These considerations do not, of course, imply that standard quality control chart methods are valueless. On the contrary, their application has resulted in the elimination of thousands of quality problems and has produced improvement in a host of industrial processes. Rather, this is a further demonstration of the obvious fact that models must be treated as approximations: all models are wrong but some models are useful. Notice that this implies with probability 1, that no “optimal” scheme is ever *in practice* optimal. What we should aim at therefore are schemes which are *robust* and *good* over a wide range of circumstances. We say good as well as robust because we could claim robustness for a system that was universally equally bad.

Now, while for the detection of assignable causes we need only to approximate fairly short run behavior, the proportion of defective articles depends on capability indices that reflect long run process behavior for which the short-term fixed mean approximation is inadequate. When steering a boat we should certainly not try to allow for every wave. Nevertheless, if we want to stay off the rocks, it will be necessary to compensate for the current.

Thus while continuing to use standard methods of quality monitoring for the assignment and elimination of special causes we will frequently need to use feedback adjustment as well.

### Feedback Adjustment in the SPC Context

Using feedback adjustment it ought to be possible to remove a considerable part of the systematic drift which is allowed for in the Six Sigma specification. This could make possible tighter specification limits and production of an even better product. The importance of developing feedback control methods suitable for use in the SPC environment has been recognized for some considerable time (See

e.g. Box and Jenkins, 1968; MacGregor, 1987; Box and Kramer, 1992; Box, Jenkins and Reinsel, 1994; Box and Luceño, 1997; Box, 1998.)

### Two Examples

In the context of the Six Sigma model, we illustrate two simple possibilities for process adjustment with an example.

*Repeated Adjustment:* Figure 1(a) shows a constructed series representing some quality characteristic  $y$  for a process that drifts. This series has an overall sample standard deviation  $\hat{\sigma}_y$  of 1.25. As explained in more detail in the Appendix, it has been obtained by adding a drift component  $d_t$  with  $\hat{\sigma}_d=0.76$  to a random noise component  $a$  with  $\sigma_a=1.0$ . The  $6\sigma$  limits, shown by bold lines in the figures, are based on  $\sigma_a=1$ . This will be a slight underestimate of the *short term* standard deviation of  $y$  obtained, for example, by using the rational subgroup method or the moving range method. The dotted lines indicate  $3\sigma$  limits. It should be noted that the standard deviation, 0.76 of the drift component we have used, is considerably *less* than  $\sigma_d = 1.50$  allowed for in the  $6\sigma$  specification. A robust and good feedback adjustment procedure is to compensate for only a proportion  $G$  of each deviation from target. A value for  $G$  in the neighborhood of 0.2 – 0.4 is frequently effective and  $G = 0.2$  is used in this illustration. Figure 1(b) shows the adjustments that have been made and Figure 1(c) the process output after repeatedly applying such adjustment. As shown in Figure 1(c) the sample standard deviation of the adjusted process  $\hat{\sigma}_e=1.02$  is not much greater than that of the random component. It can be shown (B.L.) that repeatedly applying this simple procedure would be equivalent to continually applying an *overall* correction to the uncontrolled process which is an exponentially weighted moving average (EWMA) of past data with smoothing constant  $1-G$  (0.8 in this example.). Figure 1(b) shows that this adjustment “filter” has produced a very good estimate of the drift which is automatically removed by the control scheme.

*Feedback Adjustment Using a Bounded Chart:* It is sometimes inconvenient to make repeated adjustments in the manner described above. In this case a *bounded adjustment* scheme may be used. It turns out that a scheme of this kind which is robust and good may be obtained as follows: action is taken only when, at time  $t$ , an EWMA  $\tilde{y}_t$  of present and past deviations from target falls outside tabled limits denoted by  $\pm L$ . The current EWMA  $\tilde{y}_t$  is most easily obtained by continually updating the previous EWMA  $\tilde{y}_{t-1}$  as soon as a new data value  $y_t$  is available using the well-known recursive formula:

$$\tilde{y}_t = Gy_t + (1 - G)\tilde{y}_{t-1}$$

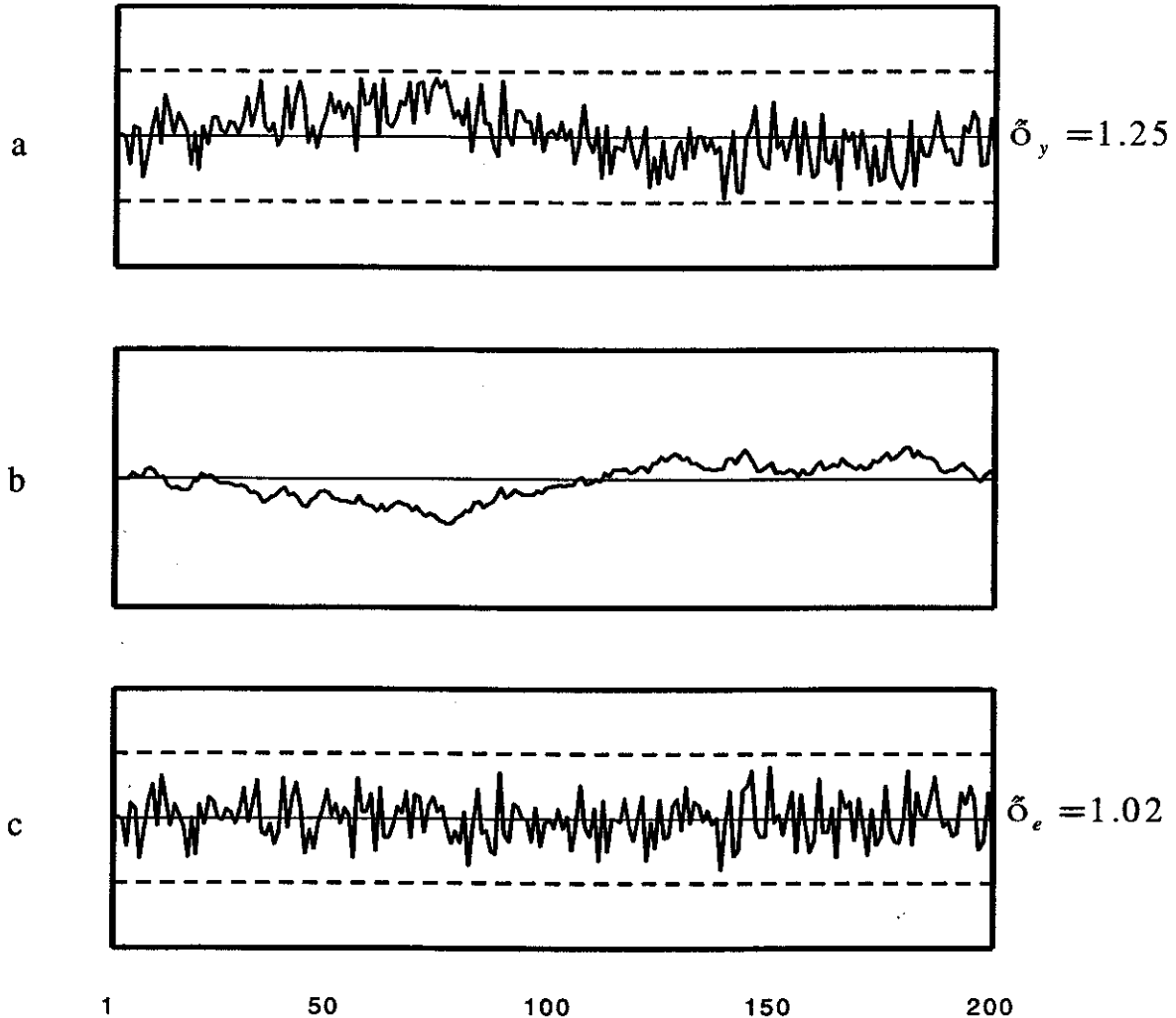


Figure 1. Performance of Repeated Adjustment Scheme: a) A process subject to drift; b) Adjustments by repeated control scheme; c) Controlled process output

or with  $G = 0.2$ :

$$\tilde{y}_t = 0.2y_t + 0.8\tilde{y}_{t-1}$$

Figure 2 illustrates this mode of control using the same constructed data as in 2(a). Figure 2(b) shows the adjustments made using  $L = \pm 1$ . (In general, the process would not be adjusted until the EWMA showed a deviation from target of  $L\sigma_a$ .) Figure 2(c) shows the controlled process output after these adjustments. The standard deviation  $\hat{\sigma}_e = 1.08$  for the process output is now slightly higher than that for the repeated scheme. These calculations are very simple and need not be very precise (see B.L.). They can be made by eyeballing an interpolation 0.2 of the distance between  $\tilde{y}_{t-1}$  and  $y_t$ , or if desired from a more formal chart, a pocket calculator, or a process computer. In figures 1 and 2 the bold lines are the  $\pm 6\sigma$  limits and the dotted lines the  $\pm 3\sigma$  limits.

\* The model on which these tables are based is slightly different from the that used in the appendix for generating the data. However, robustness properties should ensure that the tables still provide a reasonable approximation.

### Conclusion

You can of course delve much more deeply into this topic. In particular it is shown in B.L. how such methods are derived, why the procedures described are both robust and good, and when somewhat more sophisticated methods might be needed. On specific but reasonable assumptions, the continuous feedback adjustment scheme described here is designed to produce small output variation while requiring only small sized adjustments. The bounded scheme is designed to give the longest possible intervals between needed adjustments while keeping the output standard deviation as small as possible. Tables\* are provided which show alternative possibilities for various choices of the limits  $\pm L$ . For instance, the tables indicate that for the value  $L = \pm 1.0$  used in the example the average adjustment interval (AAI) — the long run average interval between needed

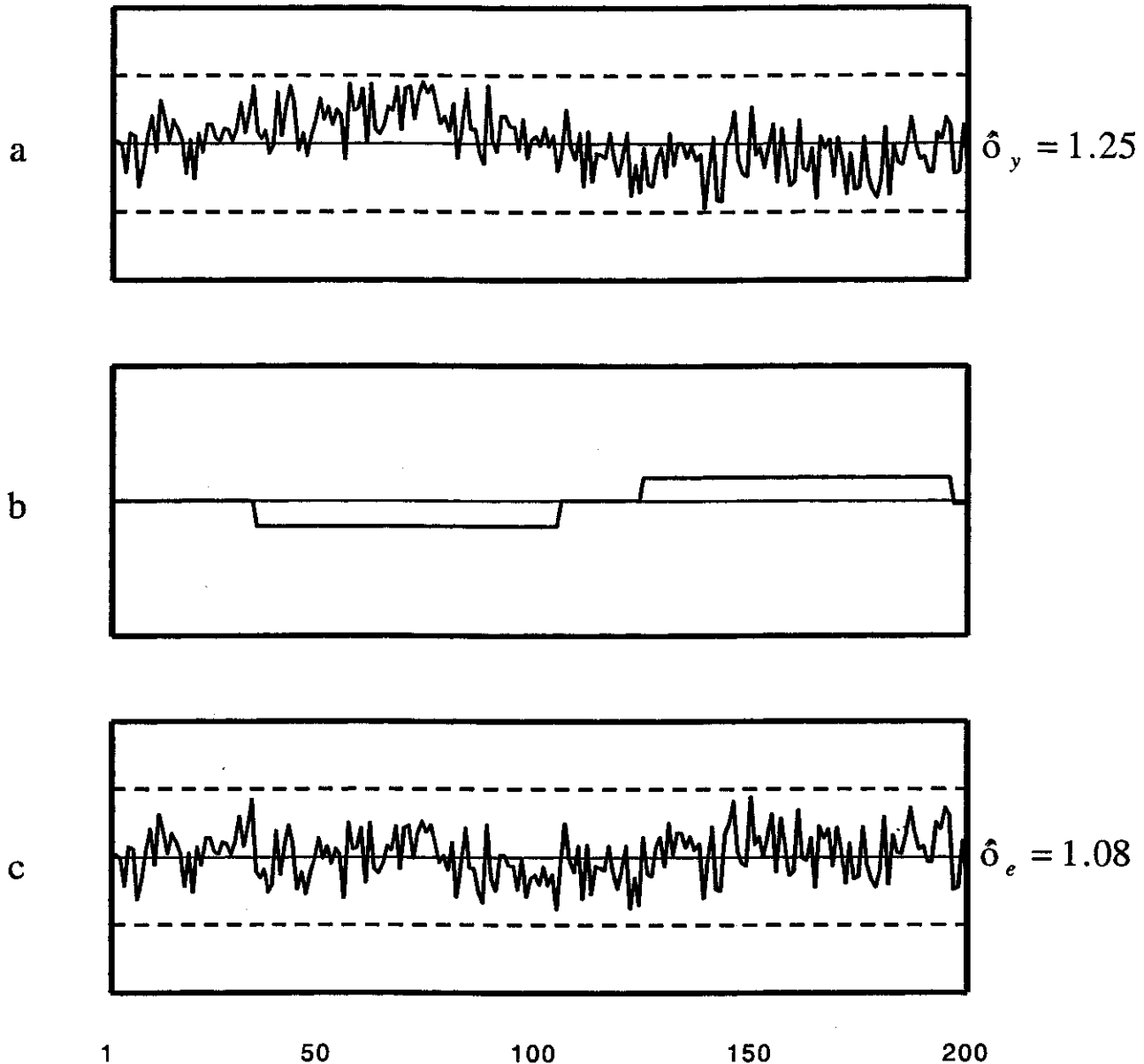


Figure 2. Performance of Bounded Adjustment Scheme: a) Process subject to drift (same as Fig. 1a); b) Adjustments by bounded control scheme; c) Controlled process output

adjustments — would be about 31 with an output standard deviation of about 1.09. This is in reasonable agreement with what we see in Figures 2b and 2c. If we had used  $L = 1.5$  (i.e. if we had set limits at  $\pm 1.5$ ) the tables indicate that the AAI would have increased to about 66 with an increased output standard deviation of about  $1.19\sigma$ . The theory underlying the two kinds of schemes is the same. Indeed, the repeated adjustment procedure is obtained if  $L$  is set equal to zero in the bounded adjustment scheme. Of course not all feedback adjustment problems can be dealt with in this simple manner. But, schemes of this kind do seem to be widely applicable and they provide an excellent base from which to practice and study process adjustment for SPC.

### Appendix: Some Details of the Calculations

**White Noise:** In this article the term “white noise” is used to mean a series of independently distributed random variables which are (roughly) normally distributed with fixed standard deviation and mean zero.

**Generation of the constructed “process” series in Figures 1(a) and 2(a):** Values  $y_t$  for the series in Figure 1(a) and 2(a) simulating a drifting process were generated by adding a drift component  $d_t$  to a random component  $a_t$  so that  $y_t = d_t + a_t$ . The component representing a slow but random drift was generated by the autoregressive model:

$$d_t = 0.99d_{t-1} + \alpha_t$$

with an observation from a white noise series with standard deviation of  $\sigma_\alpha$ . It is shown in time series texts that the relation between  $\sigma_\alpha$  and  $\sigma_d$  is then

$$\sigma_d = (1 - 0.99^2)^{-\frac{1}{2}} \sigma_\alpha$$

Thus, for example, by setting  $\sigma_\alpha = 0.21$  we could generate a drift series with  $\sigma_d = 1.5$  as allowed for in the  $6\sigma$  specification. However, to show the great sensitivity of these feedback adjustment schemes, we have employed a less pronounced drift component with  $\sigma_d$  of only 0.76. The random component, ( $\alpha_t$  say) is a white noise series with standard deviation equal to one. This would represent the theoretical behavior of an "in control" process having fixed mean and no drift. For the data of our example the resulting value for  $\hat{\sigma}_y$  is 1.25. The white noise component is of course irremovable by control but represents the best output (in practice unattainable) that any control scheme could accomplish.

*Choice of G:* An "ad hoc" value of  $G = 0.2$  was used in these examples. This choice is not very critical but if desired a basis for the choice of  $G$  (or for choosing between any alternative control schemes) is as follows. Using the series from an uncontrolled process (e.g. as in Figure 1(a)), a series of computer runs is made employing adjustment schemes for various choices of  $G$ . This shows, for each scheme, about how large the resulting output standard deviation would be and how much manipulation would be necessary. An informed choice may then be made taking into account the context in which the scheme will be used. The same procedure can be used to test any alternative method of control. Sometimes uncontrolled process data is not available. However, as a preliminary, it will often be possible to reconstruct the "uncontrolled" series. This can be done by making appropriate allowance for the changes that are known to have been made and thus to reconstruct the "uncontrolled" series.

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