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$\overline{X}$ Charts With Variable Parameters

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ABSTRACT

The idea of varying the $\overline{X}$ chart parameters has been explored extensively in recent years. Basically, the $\overline{X}$ value establishes if the control should be relaxed or not. When $\overline{X}$ falls near the target the control is relaxed because one will wait more to take the next sample and/or the next sample will be smaller than usual. When $\overline{X}$ falls far from the target but not in the action region the control is tightened because one will wait less to take the next sample and/or the next sample will be larger than usual. In this paper, we extend this study to consider the action limits variable too. The idea is to draw the action limits wider than usual when the control is relaxed and narrower than usual when the control is tightened. This new feature makes the $\overline{X}$ chart comparable with the CUSUM and EWMA schemes in terms of the speed in which they detect small shifts in the process mean.

KEYWORDS: Control charts; Statistical process control; Variable parameters.

$\overline{X}$ Charts With Variable Parameters

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The idea of varying the $\overline{X}$ chart parameters has been explored extensively in recent years. Basically, the $\overline{X}$ value establishes if the control should be relaxed or not. When $\overline{X}$ falls near the target the control is relaxed because one will wait more to take the next sample and/or the next sample will be smaller than usual. When $\overline{X}$ falls far from the target but not in the action region the control is tightened because one will wait less to take the next sample and/or the next sample will be larger than usual. In this paper, we extend this study to consider the action limits variable too. The idea is to draw the action limits wider than usual when the control is relaxed and narrower than usual when the control is tightened. This new feature makes the $\overline{X}$ chart comparable with the CUSUM and EWMA schemes in terms of the speed in which they detect small shifts in the process mean.

1. INTRODUCTION

The work of Reynolds, Amin, Arnold, and Nachlas (1988) introduced the idea of varying the $\overline{X}$ chart parameters as a function of what is observed from the process. The interval between samples was the first parameter to be considered variable (see Reynolds, Amin, Arnold, and Nachlas (1988), Runger and Pignatiello (1991), Amin and Miller (1993), Runger and Montgomery (1993), Reynolds, Arnold and Baik (1996) and Reynolds (1996)). The variable sampling interval feature was extended to CUSUM and EWMA charts (see Reynolds, Amin and Arnold (1990) and Saccucci, Amin and Lucas (1992)). Recently Baxley (1995) presented an application of EWMA chart with variable sampling intervals at Monsanto's nylon fiber plant in Pensacola, Florida.

The size of the samples was the second parameter to be considered variable (see Prabhu, Runger and Keats (1993) and Costa (1994)). Subsequently, both parameters (sample size and sampling interval) were made variable (see Prabhu, Montgomery and Runger (1994) and Costa (1995)). Costa (1996) provided an illustrative example to motivate the use of the joint $\overline{X}$ and R charts with variable sample sizes and sampling intervals.

This article studies the $\overline{X}$ chart when all design parameters are variable (Vp $\overline{X}$ chart): the sample size n, the sampling interval h and the k factor used in determining the width of the action limits. These three parameters vary between minimum and maximum values as a function of the last sample point position. If this point falls in the warning region it is reasonable to tighten the control:

a) waiting less to take the next sample (minimum h);

b) increasing the size of the next sample (maximum n) and

c) plotting the next sample point on the $\overline{X}$ chart with narrow action limits (minimum k).

On the other hand, if the last sample point falls in the central region it is reasonable to relax the control:

a) waiting more to take the next sample (maximum h);

b) decreasing the size of the next sample (minimum n) and

c) plotting the next sample point on the $\overline{X}$ chart with wide action limits (maximum k).

The organization of this paper is as follows. A general description of the Vp $\overline{X}$ chart is presented. The expectation and the standard deviation of the time to detect process mean shifts (with a Vp $\overline{X}$ chart) are obtained. Finally, the Vp $\overline{X}$ chart is compared with several other charts, including CUSUM and EWMA, in terms of their speed in detecting off-target conditions.

2. DESCRIPTION OF THE $\overline{X}$ CHART WITH VARIABLE PARAMETERS

Throughout this article, it is assumed that the $\overline{X}$ chart with variable parameters is employed to monitor a process where the observations from the quality characteristic of interest X are normally distributed with mean μ and known variance σ^2. The process is considered to start with the mean at target ($\mu = \mu_0$) but at some random time in the future the mean shifts from μ_0 to $\mu_1 = \mu_0 \pm \delta\sigma$, $\delta > 0$. Before the shift, the process is considered to be in a state of statistical control (shortly defined by in-control state). The particular case, when the process operates with the mean off-target ($\mu = \mu_1$) since the beginning, will be studied too.

When the $\overline{X}$ chart is used for monitoring the process a sample of size n_0 is randomly chosen every h_0 hours. Then, the $\overline{X}$ values from the samples are plotted on the control chart with upper and lower control limits given by $\mu_0 \pm k_0\sigma/\sqrt{n_0}$.

The Vp $\overline{X}$ chart is a modification of the $\overline{X}$ chart in which the design parameters n, h and k vary between two values in function of the most recent process information. This way, the position of each sample point on the chart establishes the size of the next sample and the instant of its sampling (see Figure 1). If the sample point falls in the warning region the next sample should be large, that is $n_2 > n_0$, and it should be sampled after a short time interval, that is $h_2 < h_0$. On the other hand, if the sample point falls in the central region the next sample should be small, that is $n_1 < n_0$, and it should be sampled after a long time interval, that is $h_1 > h_0$. Moreover, the $\overline{X}$ values should be plotted in a chart with warning and action limits given by $\mu_0 \pm w_i\sigma/\sqrt{n_i}$ and $\mu_0 \pm k_i\sigma/\sqrt{n_i}$ respectively, where i=1 (i=2) if $\overline{X}$ comes from the small (large) sample, $k_1 > k_0 > k_2$ and $w_1 > w_2$.

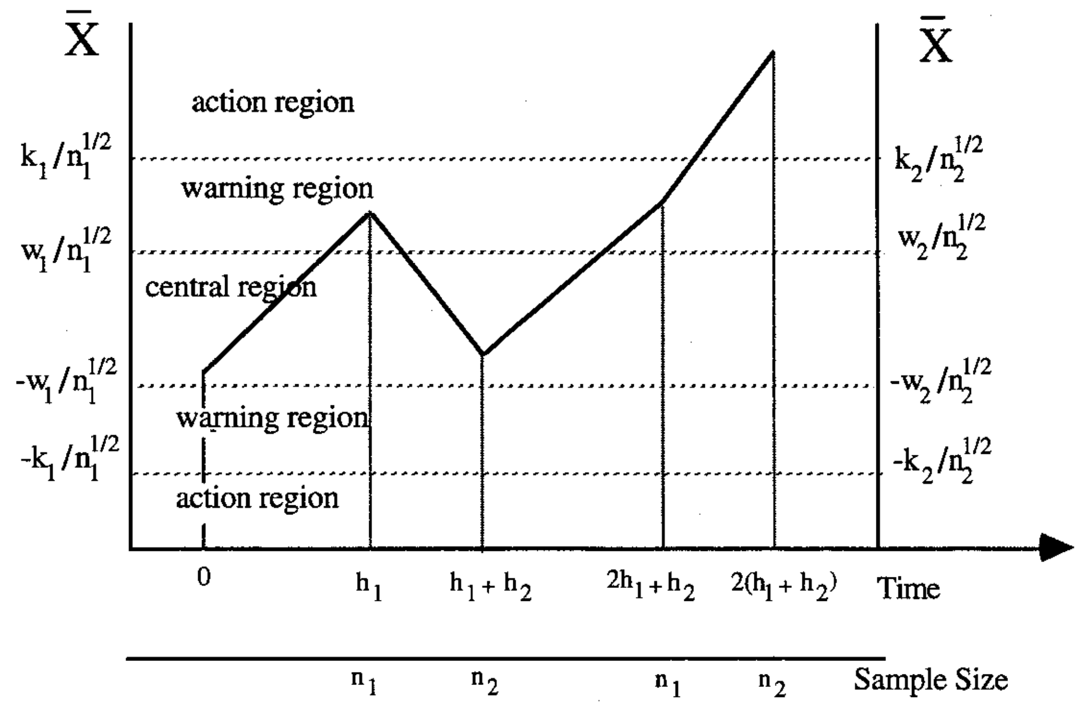

Figure 1. *Vp $\overline{X}$ Chart*.

To avoid the use of two charts, one for small samples and other for large samples, one can construct the Vp $\overline{X}$ chart with two scales, one on the left side and other on the right, as illustrated on Figure 1. Then, the $\overline{X}$ values from small samples can be plotted on the chart considering the left scale and the $\overline{X}$ values from large samples can be plotted on the chart considering the right scale. For simplicity, the $\overline{X}$ chart limits on Figure 1 were obtained making $\mu_0 = 0$ and $\sigma = 1.00$.

Likewise the $\overline{X}$ chart, the Vp $\overline{X}$ chart produces a signal when a sample point falls in the action region. If the process is operating properly with $\mu = \mu_o$ this signal is a false alarm.

The size of the first sample that is taken from the process when it is just starting, or after a false alarm, is chosen at random. If the sample was chosen to be large (small) it should be sampled after a short (long) time interval. During the in-control period all samples, including the first one, have probability p_o of being small and $(1-p_o)$ of being large, where

$$\begin{aligned} p_0 &= Pr[\,|Z|< w_1 \,|\, |Z|< k_1\,] \\ &= Pr[\,|Z|< w_2 \,|\, |Z|< k_2\,] \end{aligned} \qquad (2.1)$$

and $Z \sim N(0,1)$.

3. PERFORMANCE MEASURE

The speed with which a control chart detects process mean shifts measures its statistical efficiency. When the interval between samples is not fixed the speed is measured by the ATS (Average Time to Signal) or AATS (Adjusted Average Time to Signal). The first one is adopted when the process operates with the mean off-target $(\mu = \mu_1)$ since the beginning. This way, the ATS is the average time from the start of the process to the time when the chart signals. The second one is adopted when the process starts out with $\mu = \mu_0$ and then shifts from μ_0 to μ_1 at some random time in the future. The AATS is the average time from the process mean shift until the chart produces a signal.

The ATS for the Vp $\overline{X}$ chart depends on the size of the first sample. When the first sample is the small one, the ATS is obtained considering the number of points M_1 in the central region before signal. M_1 is distributed geometrically with parameter $1 - p_1$, where p_1 is the conditional probability to obtain another point in the central region given the current sample point belongs to the central region, that is,

$$p_1 = p_{11} + p_{12} \sum_{i=1}^{\infty} p_{22}^{i-1} p_{21} \qquad (3.1)$$

where

$$p_{11} = Pr[\,|y|< w_1 \,|\, y \sim N(\delta\sqrt{n_1},1)]$$

$$p_{12} = Pr[w_1 < |y| < k_1 \,|\, y \sim N(\sqrt{n_1},1)]$$

$$p_{22} = Pr[w_2 < |y| < k_2 \,|\, y \sim N(\sqrt{n_2},1)]$$

$$p_{21} = Pr[\,|y|< w_2 \,|\, y \sim N(\delta\sqrt{n_2},1)].$$

The time from the start of the process to the time when the chart signals is given by

$$T_1 = \sum_{i=1}^{M_1} L_i . \qquad (3.2)$$

The variables L_i, are independents and identically distributed as a random variable Y_1, where Y_1 is the length of time to obtain another point outside the warning region, given the current point belongs to the central region. The distribution of Y_1 is

$$Pr[Y_1 = h_1] = 1 - p_{12}$$

and

$$Pr[Y_1 = h_1 + ih_2] = p_{12} p_{22}^{i-1}(1 - p_{22}),$$
$$i = 1, 2, \ldots.$$

Hence,

$$E(Y_1) = h_1 + h_2 p_{12} / (1 - p_{22}).$$

If the first sample is the small one the ATS is the expected value of T_1, which is expressed as

$$\begin{aligned} E(T_1) &= E(M_1)E(Y_1) \\ &= [h_1(1-p_{22}) + h_2 p_{12}] / Q \end{aligned}$$

where $Q = 1 - p_{11} - p_{22} + p_{11}p_{22} - p_{12}p_{21}$.

If $C_1 = Y_1 - h_1$ then $V(C_1) = V(Y_1)$ and

$E(C_1^2) = h_2^2 p_{12} E(D^2)$ where D is geometrically distributed with parameter $1 - p_{22}$. Therefore,

$$V(Y_1) = E(C_1^2) - [E(C_1)]^2$$
$$= h_2^2[p_{12}(1+p_{22}) - p_{12}^2]/(1-p_{22})^2$$

Using the fact that

$$V(T_1) = E(M_1)V(Y_1) + V(M_1)[E(Y_1)]^2,$$

it follows then that

$$V(T_1) = h_2^2[p_{12}(1-p_{12}) + p_{12}p_{22}]/Q(1-p_{22}) + $$
$$+[(1-p_{22}-Q)(1-p_{22})]\times$$
$$[h_1 + h_2 p_{12}/(1-p_{22})]^2/Q^2.$$

When the first sample is the large one, the ATS is obtained considering the number of points M_2 in the warning region before signal. M_2 is distributed geometrically with parameter $1 - p_2$, where p_2 is the conditional probability to obtain another point in the warning region given the current sample point belongs to the warning region, that is,

$$p_2 = p_{22} + p_{21}\sum_{i=1}^{\infty} p_{11}^{i-1} p_{12}. \qquad (3.3)$$

The time from the start of the process to the time when the chart signals is given by

$$T_2 = \sum_{i=1}^{M_2} O_i. \qquad (3.4)$$

The variables O_i, are independents and identically distributed as a random variable Y_2, where Y_2 is the length of time to obtain another point outside the central region, given the current point belongs to the warning region. The distribution of Y_2 is

$$\Pr[Y_2 = h_2] = 1 - p_{21}.$$

and

$$\Pr[Y_2 = h_2 + ih_1] = p_{21}p_{11}^{i-1}(1-p_{11}),$$
$$i = 1, 2, \ldots.$$

Hence,

$$E(Y_2) = h_2 + h_1 p_{21}/(1-p_{11}).$$

If the first sample is the large one the ATS is the expected value of T_2, where

$$E(T_2) = E(M_2)E(Y_2)$$
$$= [h_2(1-p_{11}) + h_1 p_{21}]/Q$$

and

$$V(T_2) = E(M_2)V(Y_2) + V(M_2)[E(Y_2)]^2$$
$$= h_1^2[p_{21}(1-p_{21}) + p_{21}p_{11}]/Q(1-p_{11}) + $$
$$+[(1-p_{11}-Q)(1-p_{11})]\times$$
$$[h_2 + h_1 p_{21}/(1-p_{11})]^2/Q^2.$$

When the first sample has probability p_0 of being small and $1 - p_0$ of being large the ATS is given by

$$ATS = E(T_1)p_0 + E(T_2)(1-p_0) \qquad (3.5)$$

To obtain the AATS, let A=length of the interval in which the shift occurs, R=time from the process shift until the next sample, S=time from the next sample after the process shift until signal and T = time from the process shift until signal. From Figure 2 one can see that T=R+S and AATS is

$$E(T) = E(R) + E(S).$$

As Reynolds, Amin, Arnold, and Nachlas (1988) we assume that the probability of the mean moving off-target in an interval between samples is proportional to the product of the length of the interval and the probability of an interval of that length occurring. This means that

$$\Pr[A = h_1] = p_0 h_1/[p_0 h_1 + (1-p_0)h_2] \qquad (3.6)$$

and

$$\Pr[A = h_2] =$$
$$(1-p_0)h_2/[p_0 h_1 + (1-p_0)h_2]. \qquad (3.7)$$

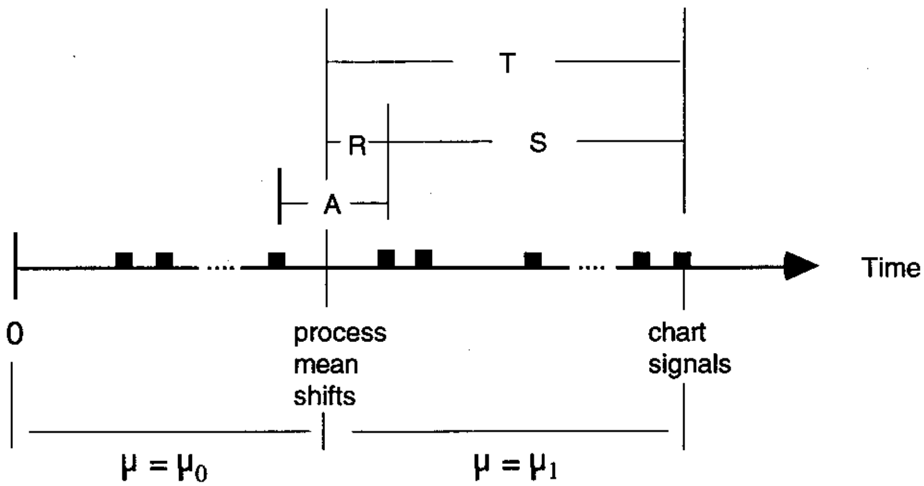

Figure 2. *Time to Detect the Shift in the Process Mean .*
■ *Points Where the Sampling is Performed.*

In addition, the time between the shift in the mean and the next sample is uniformly distributed. This means that

$$E(R|A = h_i) = h_i / 2, \qquad i = 1,2. \qquad (3.8)$$

It follows from (3.6), (3.7) and (3.8) that

$$E(R) = [p_0 h_1^2 + (1 - p_0) h_2^2] / 2 \times [p_0 h_1 + (1 - p_0) h_2].$$

The expected value of S depend on the position of the first sample point after shift (B). The probability of plotting this point in the central region ($B = B_1$), warning region ($B = B_2$) or action region ($B = B_3$) depends on the length of the interval in which the shift occurs. This leads to

$$\Pr[B = B_1] = \Pr[B = B_1|A = h_1]\Pr[A = h_1] + \Pr[B = B_1|A = h_2]\Pr[A = h_2] = p_{11}\Pr[A = h_1] + p_{21}\Pr[A = h_2],$$

$$\Pr[B = B_2] = p_{12}\Pr[A = h_1] + p_{22}\Pr[A = h_2],$$

and

$$\Pr[B = B_3] = (1 - p_{11} - p_{12}) \times \Pr[A = h_1] + (1 - p_{21} - p_{22})\Pr[A = h_2].$$

When the first sample point after shift falls in the central region $S = T_1$. When this point falls in the warning region $S = T_2$. Therefore,

$$E(S) = \Pr[B = B_1]E(T_1) + \Pr[B = B_2]E(T_2).$$

Since *R* and *S* are not independents, the appropriate expression to obtain the variance of the adjusted time to signal is

$$V(T) = E_{A\cap B}[V(T|A \cap B)] + V_{A\cap B}[E(T|A \cap B)] = E(T_1^2)\Pr[B = B_1] + E(T_2^2)\Pr[B = B_2] - E(T)^2 + [h_1^2 / 3 + E(T_1)h_1 p_{11} + E(T_2)h_1 p_{12}]\Pr[A = h_1] + [h_2^2 / 3 + E(T_1)h_2 p_{21} + E(T_2)h_2 p_{22}]\Pr[A = h_2]. \qquad (3.9)$$

4. THE VP $\overline{X}$ CHART DESIGN

The Vp $\overline{X}$ chart six parameters, (n_1, n_2), (h_1, h_2) and (k_1, k_2) will be settled with the following constrains:

$$n_1 p_0 + n_2(1 - p_0) = n_0 \qquad (4.1)$$

$$h_1 p_0 + h_2(1 - p_0) = h_0 \qquad (4.2)$$

$$\Pr[|Z|> k_1]p_0 + \Pr[|Z|> k_2](1-p_0) = \Pr[|Z|> k_0] \quad (4.3)$$

where , n_0, h_0 and k_0 are the parameters of the Shewhart standard $\overline{X}$ chart (SS $\overline{X}$ chart). Usually n_0 =3, 4 or 5, k_0 =3.00 and without losing generality one can consider h_0 =1.00. The constrains (4.1), (4.2) and (4.3) ensure that the false alarm rate is the same for both charts (Vp and SS $\overline{X}$ charts) and that, on average, the two charts require the same number of items per unit of time to be sampled while the process remains in-control.

The three constrains (4.1), (4.2) and (4.3) allow the practitioner to choose one of the pairs (n_1,n_2), (h_1,h_2) or (k_1,k_2) and one element from each remaining pair. We recommend choosing the pair (n_1,n_2), and the elements h_2 and k_1 for two reasons:

(a) the range of feasible values for n_2 and h_2 depend on the time required to sample each item;

(b) Vp chart is strongly recommended to detect small shifts in the process mean (when the shift is small ($\delta\sqrt{n_0}$ around 1.00) the chart works better with n_1=1 and k_1 enough big to make the risk of false alarms practically zero).

It follows from constrains (4.1) and (4.3) that

$$k_2 = \Phi^{-1}\{[(n_2-n_1)\Phi(k_0) - (n_2-n_0)\Phi(k_1)]/(n_0-n_1)\},$$

from (2.1) and (4.1)

$$w_i = \Phi^{-1}\{[2(n_2-n_0)\Phi(k_i)+n_0-n_1]/2\times(n_2-n_1)\}, \quad i=1,2$$

and finally from (4.3)

$$h_1 = [h_0(n_2-n_1) - h_2(n_0-n_1)]/(n_2-n_0).$$

When $n_0 = n_1 = n_2$ follows from constrains (4.2) and (4.3) that

$$k_2 = \Phi^{-1}\{[(h_1-h_2)\Phi(k_0) - (h_0-h_2)\Phi(k_1)]/(h_1-h_0)\},$$

and from (2.1) and (4.2)

$$w_i = \Phi^{-1}\{[2(h_0-h_2)\Phi(k_i) + h_1-h_0]/2(h_1-h_2)\}, \quad i=1,2.$$

5. COMPARING CHARTS

Once the Vp $\overline{X}$ chart was designed to match to the SS $\overline{X}$ chart (in terms of the in-control performance) one can compare their speed in detecting process mean shifts. Table 1 provides the adjusted average time to signal (AATS) for several Vp $\overline{X}$ charts designed to match to a SS $\overline{X}$ chart with h_0 =1.00, k_0 =3.00 and n_0=4. The Vp $\overline{X}$ chart parameters were selected in order to optimize its efficiency in detecting small shifts for three combinations of n_2 and h_2 values: (n_2, h_2)= (8,0.05), (12,0.10) and (16,0.25). Table 1 shows the substantial improvement on the $\overline{X}$ chart performance when the design parameters *n*, *h* and *k* are variable. In addition, Table 2 shows that the $\overline{X}$ chart with variable parameters promotes the reduction of both: the expectation and the standard deviation of *T* (adjusted time to signal).

When $k_0 = k_1 = k_2$ the Vp $\overline{X}$ chart is called $\overline{X}$ chart with variable sample sizes and sampling intervals (VSSI $\overline{X}$ Chart), when $k_0 = k_1 = k_2$ and $h_0 = h_1 = h_2$ the Vp $\overline{X}$ chart is called $\overline{X}$ chart with variable sample size (VSS $\overline{X}$ Chart). Finally, when $k_0 = k_1 = k_2$ and $n_0 = n_1 = n_2$ the Vp $\overline{X}$ chart is called $\overline{X}$ chart with variable sampling intervals (VSI $\overline{X}$ Chart). Costa (1995) compared the SS, VSI, VSS and VSSI charts in terms of the AATS.

It seems reasonable to compare the Vp $\overline{X}$ chart with the CUSUM and EWMA charts because we are suggesting the use of this chart for detecting small process mean shifts. Table 3 provides the average time to signal (ATS) for the Vp $\overline{X}$ charts designed to match to the CUSUM schemes with parameters *(k,h)*=(0.25,7.27), (0.50,4.39), (0.75,3.08) and the EWMA scheme with parameters (λ, h)=(0.15,0.76). From Table 3, one can see that Vp $\overline{X}$ chart is comparable with the CUSUM and EWMA charts in terms of the speed they detect small shifts in the process mean. The CUSUM and EWMA charts were designed to detect small shifts (δ=1.00, see Gan (1991)).

Another way to improve the $\overline{X}$ chart performance consists in introducing supplementary run rules. Let $T(k,m,a,b)$ denote the run rule that signals if k of the last m standardized sample means fall in the interval (a,b) for $a<b$, and consider the three runs rules

$$C_1 = \{T(1,1,-\infty,-3), T(1,1,3,\infty)\}$$

$$C_2 = \{T(2,3,-3,-2), T(2,3,2,3)\}$$

$$C_3 = \{T(4,5,-3,-1), T(4,5,1,3)\}.$$

The chart C_{ijk} denotes the $\overline{X}$ chart with runs rules i, j, and k implemented. The Table 4 compares the Vp $\overline{X}$ chart with the C_{12}, C_{13} and C_{123} charts. The FORTRAN program written by Champ and Woodall (1990) was used to calculate the ATS for the $\overline{X}$ chart with supplementary run rules. One can see from Table 4 that the Vp $\overline{X}$ chart is always quicker than the $\overline{X}$ chart with supplementary run rules in detecting process mean shifts.

Table 1. Values of AATS for the $V_p\,\overline{X}$ Chart

			$\delta\sqrt{n_0}$								
(n_1,n_2)	(h_1,h_2)	(k_1,k_2)	0.0	0.50	0.75	1.00	1.25	1.50	2.00	3.00	4.00
(4, 4)	(1.00,1.00)	(3.00,3.00)	370	155	80.7	43.4	24.5	14.5	5.80	1.50	0.69
(1, 8)	(1.71,0.05)	(2.73,6.00)	370	87.7	32.1	12.6	5.88	3.45	2.07	1.39	1.10
(1,12)	(1.34,0.10)	(2.58,6.00)	370	65.8	22.3	8.99	4.76	3.25	2.21	1.42	1.04
(1,16)	(1.19,0.25)	(2.47,6.00)	370	54.1	18.2	7.95	4.75	3.54	2.52	1.59	1.16
(1, 8)	(1.71,0.05)	(3.00,3.00)	370	127	48.7	18.2	7.52	3.92	2.10	1.38	1.09
(1,12)	(1.34,0.10)	(3.00,3.00)	370	118	40.1	13.7	5.88	3.51	2.23	1.40	1.02
(1,16)	(1.19,0.25)	(3.00,3.00)	370	111	34.7	11.7	5.57	3.72	2.52	1.56	1.10
(1, 8)	(1.00,1.00)	(3.00,3.00)	370	139	59.9	25.9	12.2	6.54	2.76	1.30	1.04
(1,12)	(1.00,1.00)	(3.00,3.00)	370	127	47.3	18.0	8.15	4.62	2.46	1.54	1.25
(1,16)	(1.00,1.00)	(3.00,3.00)	370	117	39.0	14.0	6.62	4.18	2.63	1.76	1.38
(4, 4)	(2.00,0.05)	(3.00,3.00)	370	141	65.3	30.1	14.2	7.00	2.28	1.06	0.99
(4, 4)	(2.00,0.10)	(3.00,3.00)	370	141	66.0	30.7	14.6	7.35	2.45	1.08	0.97
(4, 4)	(2.00,0.25)	(3.00,3.00)	370	143	68.1	32.5	16.1	8.43	2.98	1.15	0.95

Table 2. Values of the Standard Deviation of the Adjusted Time to Signal for the $V_p\,\bar{X}$ Chart

			$\delta\sqrt{n_0}$								
(n_1,n_2)	(h_1,h_2)	(k_1,k_2)	0.0	0.50	0.75	1.00	1.25	1.50	2.00	3.00	4.00
(4, 4)	(1.00,1.00)	(3.00,3.00)	370	155	80.7	43.4	24.5	14.5	5.80	1.44	0.55
(1, 8)	(1.71,0.05)	(2.73,6.00)	370	87.0	31.5	12.3	5.64	3.28	1.90	1.17	0.81
(1,12)	(1.34,0.10)	(2.58,6.00)	370	65.2	21.8	8.75	4.64	3.17	2.12	1.25	0.80
(1,16)	(1.19,0.25)	(2.47,6.00)	370	53.6	17.9	7.86	4.72	3.50	2.44	1.39	0.85
(1, 8)	(1.71,0.05)	(3.00,3.00)	370	127	48.3	17.8	7.28	3.73	1.93	1.17	0.81
(1,12)	(1.34,0.10)	(3.00,3.00)	370	118	39.8	13.4	5.72	3.40	2.55	1.24	0.80
(1,16)	(1.19,0.25)	(3.00,3.00)	370	111	34.5	11.6	5.48	3.65	2.43	1.38	0.85
(1, 8)	(1.00,1.00)	(3.00,3.00)	370	139	60.0	26.0	12.2	6.44	2.48	0.99	0.70
(1,12)	(1.00,1.00)	(3.00,3.00)	370	127	47.4	18.2	8.26	4.61	2.30	1.22	0.82
(1,16)	(1.00,1.00)	(3.00,3.00)	370	117	39.1	14.2	6.80	4.25	2.57	1.45	0.93
(4, 4)	(2.00,0.05)	(3.00,3.00)	370	141	65.3	30.0	14.1	6.90	2.05	0.66	0.59
(4, 4)	(2.00,0.10)	(3.00,3.00)	370	141	66.0	30.6	14.6	7.21	2.17	0.68	0.60
(4, 4)	(2.00,0.25)	(3.00,3.00)	370	143	68.1	32.5	16.0	8.22	2.62	0.76	0.64

Table 3. Values of the ATS for the $V_p\,\bar{X}$ Chart, CUSUM Scheme and EWMA Scheme

			δ								
$V_p\,\bar{X}$ Chart			0.0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
(n_1,n_2)	(h_1,h_2)	(k_1,k_2)									
(1, 6)	(1.22,0.10)	(2.06,5.00)	250	112	31.4	11.5	6.18	4.40	3.56	3.08	2.76
(1,12)	(1.19,0.25)	(1.89,5.00)	250	79.4	21.7	11.6	8.92	7.56	6.64	6.00	5.56
CUSUM(0.25,7.27)			250	70.6	25.9	14.9	10.4	8.01	6.53	5.52	4.80
CUSUM(0.50,4.39)			250	95.6	30.8	14.7	9.16	6.58	5.14	4.22	3.60
CUSUM(0.75,3.08)			250	121	41.6	18.0	9.96	6.56	4.84	3.83	3.18
EWMA(0.15,0.76)			250	80.9	26.9	13.6	8.75	6.39	5.05	4.18	3.59

Table 4. Values of ATS for the $V_p \overline{X}$ Chart and $\overline{X}$ Chart with Supplementary Run Rules

$V_p \overline{X}$ Chart			δ								
(n_1,n_2)	(h_1,h_2)	(k_1,k_2)	0.0	0.50	0.75	1.00	1.25	1.50	2.00	3.00	4.00
(1, 8)	(1.68,0.10)	(2.56,5.00)	225	59.4	23.1	9.68	4.74	2.89	1.80	1.35	1.18
	C_{12}		225	77.7	37.9	20.0	11.6	7.30	3.69	1.68	1.18
(1, 8)	(1.68,0.10)	(2.46,5.00)	166	46.1	19.2	8.43	4.35	2.76	1.78	1.34	1.18
	C_{13}		166	46.2	22.4	12.7	8.17	5.86	3.68	1.89	1.19
(1, 8)	(1.68,0.10)	(2.67,5.00)	133	38.6	16.8	7.65	4.10	2.68	1.77	1.34	1.18
	C_{123}		133	38.6	19.2	11.0	7.10	5.08	3.14	1.67	1.18

6. CONCLUSION

Costa (1995) and Prabhu, Montgomery and Ruger (1994) showed that the $\overline{X}$ chart performance improves substantially when the sample size n, and the sampling interval h, vary between two values based on the location of the last sample point on the chart. With a minimum additional effort one can also vary the last design parameter k between two values. This new feature makes the $\overline{X}$ chart more sensitive in detecting small shifts in the process mean (comparable with the CUSUM and EWMA sensitivity).

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