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A Discussion of Scientific Methods for Setting Manufacturing Tolerances

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ABSTRACT

Several traditional and newer techniques for setting manufacturing tolerances are discussed. The traditional methods include worst case, statistical case, and proportional and constant factor scaling. Newer methods, such as Optimization and Monte Carlo Simulation are described more briefly. The Estimated Mean Shift model is included as a method for setting tolerances more realistically, while at the same time improving the communication between design and manufacturing departments. Additionally, some techniques are described for setting initial tolerances when little or no data or tables are available. Three tolerancing examples are included.

KEYWORDS: Constant precision factor scaling; Estimated mean shift model; Linear optimization, Loss Function; Monte Carlo simulation; Natural tolerance limits; Proportional factor scaling; Specifications; Statistical-case tolerancing; Tolerance limits; Worst-case tolerancing.

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A Discussion of Scientific Methods for Setting Manufacturing Tolerances

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Several traditional and newer techniques for setting manufacturing tolerances are discussed. The traditional methods include worst case, statistical case, and proportional and constant factor scaling. Newer methods, such as Optimization and Monte Carlo Simulation are described more briefly. The Estimated Mean Shift model is included as a method for setting tolerances more realistically, while at the same time improving the communication between design and manufacturing departments. Additionally, some techniques are described for setting initial tolerances when little or no data or tables are available. Three tolerancing examples are included.

INTRODUCTION

When the quality of a company's product is competitive in the market, it becomes increasingly important for its designers to develop the next generation of products before the competition and with better quality. Quality improvement tools can help in this respect, shortening the development cycle by avoiding ambiguities and aimless arguments, essentially narrowing the focus of the designers.

Some of the quality improvement tools for product development include experimental design, statistical process control, robust parameter design, and statistical tolerance setting. The more information the design engineer can obtain early in the development process, the better. This enables informed decision-making early on and avoids problems and added cost downstream.

BACKGROUND

Traditional tolerance setting was less than scientific. According to Juran (1988), engineers typically set mechanical tolerances:

- by precedent, which actually creates a bias toward tighter tolerances
- by bargaining with manufacturing, which is easier than making a thorough investigation
- from tolerance systems that include tables of shop standards for basic parts

The use of tolerance systems formulated by the company, industry, etc. can be helpful and time-saving provided a thorough study has been undertaken to assure the relevance and accuracy of the standards.

Complicating the application of the first two less-than-scientific methods has been the historical lack of cooperation between design and manufacturing departments. Manufacturing engineers often blame designers for setting tolerances tighter than necessary to achieve the desired performance, causing unnecessary costs in manufacturing. Designers traditionally had responsibility for assuring that the product was fit for use but little or no responsibility for assuring its economical manufacture. When the resulting design tolerances caused trouble in production they were violated by manufacturing in order to meet delivery. After it appeared to the manufacturing engineers that these products performed satisfactorily, they distrusted other design tolerances. When the designers learned of this they set the tolerances tighter to compensate, hence a vicious cycle.

Requiring the manufacturing and design engineers to set tolerances scientifically and cooperatively would alleviate much of this problem. The scientific approaches to setting tolerances require the engineers' knowledge of the production process capabilities and often the cost of precision as well. Forcing this type of information upstream in the development cycle could certainly lead to improved decision-making early on.

This paper discusses several scientific methods for setting tolerances. Basic models for setting tolerances on interchangeable parts are described in the first section, and the more advanced in the second. The third section discusses how an engineer can set initial tolerances on a part when tables and previous data are unavailable. The paper concludes with two examples of setting tolerances on interchangeable parts. For a more detailed discussion of several of these techniques, Chase and Greenwood (1988) is an excellent resource.

TERMINOLOGY

There are some terms that might seem interchangeable but aren't, and others that seem distinct, but we will actually use them equivalently. We'll attempt to clarify this now.

According to Enrick (1977), *specifications* are specific instructions established to maintain quality standards, i.e. the product's performance. Since there is variability in materials, processes, methods, and products, each specification must have a *tolerance* to quantify acceptable amounts of variation around its target value.

Tolerance limits are set by the designer, customer, etc., regardless of the capability of the process. However, Banks (1989) defines *natural tolerance limits* to be $\pm 3\sigma$ from the target value, where σ is based on the actual capability of the process. This is an important distinction.

Tolerance analysis (a.k.a. tolerance synthesis) is the task of setting a tolerance on a final assembly (sum of parts) based on the natural tolerances of the individual parts, and *tolerance allocation* is the task of distributing the assembly's overall tolerance among its parts (see Figure 1). Generally, any tools used for tolerance analysis can also be used for tolerance allocation, and vice versa; therefore, these terms are used rather loosely.

THE BASIC METHODS

In this section the simplest methods of tolerance analysis and allocation, namely the worst case and statistical case, are described. We then explain the proportional and constant precision factor calculations that are helpful in distributing a final assembly tolerance equally among the individual parts. We also describe an example of these calculations to compare the above methods. Chase and Greenwood (1987) introduced the Estimated Mean Shift Model to pull together the benefits of both statistical and worst case methods, while avoiding some of their individual drawbacks. This method is described at the end of the section.

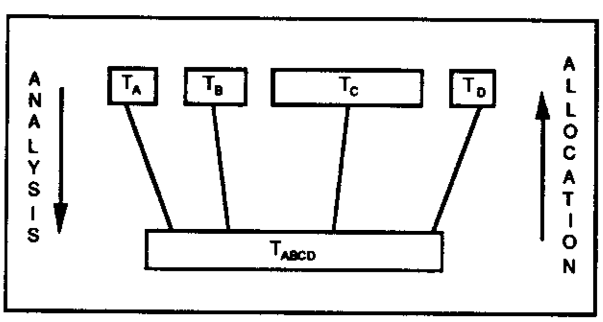

Figure 1. *Tolerance analysis vs. allocation, from Chase and Greenwood (1988).*

WORST CASE

This method is probably the most intuitive to the engineer, but it is also the most costly. For the assembly of several parts the worst case method involves a simple summation of all the individual natural tolerance ranges to obtain the full assembly tolerance. That is,

$$T_{ASSEMBLY} = \sum_{i=1}^{n} T_i$$

where T_i represents either the 6σ full-width or 3σ half-width of the distribution for part i (assuming a normally distributed dimension), and n is the total number of parts. Spotts (1983, p. 105) calls this the Arithmetic or Sure-Fit Law. From this equation one can see that tolerance analysis (using the part tolerances to calculate the assembly tolerance) would place a very large tolerance on the final assembly. On the other hand, if only the natural tolerances of the final assembly were known, its allocation would result in extremely tight tolerances on the individual parts.

STATISTICAL CASE

Tolerance setting is essentially the activity of achieving both the proper fitting of parts, as well as permitting the most economical production of the assembly. Using the worst case method is certainly safe, but it can result in exceedingly tight tolerances on the individual parts, requiring additional costs for finer grinding, increased inspection, etc.. Instead we can allow ourselves to "trust to luck" that, for example, the largest bolt of a lot will not meet with the smallest nut. In fact, if the dimensions of the individual parts can be assumed to be *independent* and roughly *normally distributed*, then there are two important statistical formulas the engineer can use to set tolerances less conservatively and more scientifically. Taken from Spotts (1983, p. 108), they are

$$\sigma_{A+B} = \sqrt{\sigma_A^2 + \sigma_B^2}$$
$$\sigma_{A-B} = \sqrt{\sigma_A^2 + \sigma_B^2}$$

where σ is the standard deviation of the dimension, and A and B are the individual parts in a two part assembly. These equations point out that regardless of whether the dimensions are summed, subtracted, or any combination thereof, the resulting variance is the square root of the sum of the individual dimensional variances. These equations produce an assembly tolerance that is much smaller than that produced with the worst case method and if the distribution mean doesn't stray off target, they are much more realistic. From these equations we can derive the tolerance formula:

$$T_{ASSEMBLY} = \left[\sum T_i^2\right]^{1/2}.$$

Note however that these tolerances must be the process-based *natural tolerances* of each component dimension in order for the statistical formulas to be valid. This is an important requirement. It is also important, though perhaps less so, that the mating parts come from different sources (e.g. sheets of metal, machines, etc.), so that the variability between them is truly independent, further validating the use of the above equations.

How much faith can we put in these statistical formulas? According to Feigenbaum (1991), many factories have conducted investigations to determine how applicable the statistical tolerance equations are to their manufacturing conditions. Those studies found that the conditions necessary for the use of the formulas are frequently met and that the statistical method can be reliably applied in tolerance analysis.

PROPORTIONAL SCALING

The engineers often find themselves in a situation where the tolerance requirements for the final assembly are given and the capabilities for the individual parts are known or can be estimated. In this case, the sum of the natural tolerances of the parts (either worst or statistical case) will most likely not be equal to the given tolerance requirement. The engineer can either allocate the excess ($T_{required} - T_{from\ sum\ of\ parts}$) back to the individual parts by loosening their tolerances or must tighten the tolerances on the parts if their sum is greater than the specification. According to Chase and Greenwood (1988), one method of doing this is to adjust the part tolerances in proportion to their tolerance widths. To obtain the proportionality factor the engineer solves one of the following equations for P:

$$T_{REQUIRED} = \sum T_{FIXED} + P \cdot \sum T_{ADJUSTABLE}$$
$$T_{REQUIRED} = \left[\sum T_{FIXED}^2 + P \cdot \sum T_{ADJUSTABLE}^2\right]^{1/2}$$

for worst and statistical case allocation, respectively. Note that the fixed tolerances refer to those parts that are purchased from an outside supplier. Hence, the engineer has no immediate control over the settings of those tolerances, and we don't multiply them by the proportionality factor. Once the proportionality factor is estimated the engineer then multiplies all the adjustable tolerances by it to obtain new tolerance widths that should provide an assembly distribution within the specifications.

CONSTANT PRECISION FACTOR

Research by Fortini (1967) has shown that parts having similar precision have equal tolerances only if the parts are similar in size, and that those tolerances are proportional to the cube root of their target values. We can use this information to introduce an additional method for setting part tolerances when the tolerance specification for the final assembly is given. The advantage of the constant precision factor over the proportional method is that the engineer does not need to know the natural tolerances of the individual parts. This can be helpful when designing new parts for which the natural tolerances are not yet known. The engineer simply allocates the tolerances according to the magnitude of the part dimension. The constant precision factor P_c is calculated from the following equations:

$$P_c = \frac{T_{REQUIRED}}{\sum D_i^{1/3}} \qquad \text{(Worst Case)}$$

$$P_c = \frac{T_{REQUIRED}}{\left[\sum D_i^{2/3}\right]^{1/2}} \qquad \text{(Statistical)}$$

where D_i is the magnitude of the dimension for part *i*. The individual part tolerances are then calculated using,

$$T_i = P_c \cdot \left(D_i^{1/3}\right).$$

There are drawbacks to this method. After the calculation the engineer still doesn't know whether the resulting tolerance settings can be met in

Table 1

Dimension	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Average	0.0505	8.0000	0.5093	0.4000	7.7110	0.4000	0.5093
Tolerances (+/–)							
Design		0.0080		0.0020	0.0060	0.0020	
Fixed	0.0015		0.0025				0.0025
Mean = A+B–C+D–E+F–G = 0.020 in.							

manufacturing. The engineer should follow up by monitoring the process capability during pilot runs.

EXAMPLE

Chase and Greenwood (1988) provide an example of the above methods by setting the tolerances for parts in a shaft and bearing assembly. A vector representation of the assembly is shown in Figure 2. The dimension of critical importance is the clearance between parts A and G. This clearance is the vector sum of all seven parts A through G. The means and tolerance half-widths of each of the parts are shown in Table 1. Parts A, C, and G are purchased from a supplier and hence we assume their tolerances are fixed.

The specification for the clearance is given as 0.020 ±0.015 inches. To achieve this in the assembly we need to set the part tolerances at the appropriate levels. Here we calculate the tolerances according to worst case proportional scaling and then by the statistical constant precision factor method.

Summing the part tolerances first by the worst case method gives,

$$T_{Clearance} = T_A + T_B + T_C + T_D + T_E + T_F + T_G = 0.0245$$

which exceeds the specification of 0.015 inches. To improve this we can reduce the individual part

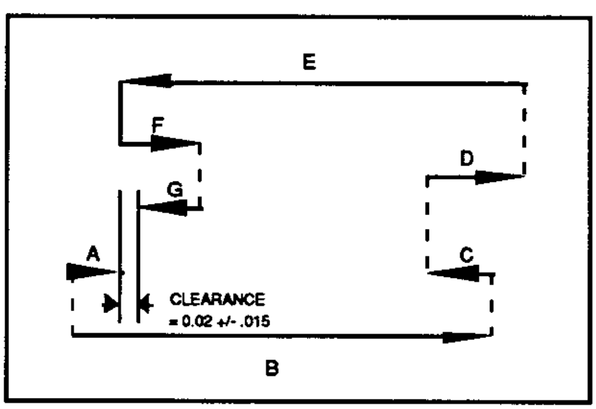

Figure 2. *Vector sum of the horizontal dimensions.*

tolerances (half-widths) by solving for the proportionality factor,

$$T_{Clearance} = T_A + T_C + T_G + P \cdot (T_B + T_D + T_E + T_F)$$

so that

$$P = 0.47222$$

Note that the tolerances for parts A, C, and G are not adjusted by the proportionality factor since their tolerances are considered fixed (purchased from a supplier). After using the proportionality factor the tolerances become,

$$T_B = 0.47222 \cdot (0.008) = 0.00378$$
$$T_D = 0.47222 \cdot (0.002) = 0.00094$$
$$T_E = 0.47222 \cdot (0.006) = 0.00283$$
$$T_F = 0.47222 \cdot (0.002) = 0.00094$$

Keeping parts B, D, E, and F within these half-widths of their respective means should ensure that the clearances in the final assemblies remain between 0.005 and 0.035 inches. However, one might argue that these tolerances are unnecessarily tight and therefore costly to manufacture. We now recalculate them based on the statistical method.

Again, when using the statistical method for summing the tolerances we assume that the dimensions are independently distributed, and that the given tolerance half-widths represent 3σ. Summing them gives,

$$\left[\Sigma T_i^2\right]^{1/2} = (.0015^2 + .008^2 + .0025^2 + .002^2 + .006^2 + .002^2 + .0025^2)^{1/2} = 0.011 \text{ in.}$$

which is much smaller than the required tolerance of 0.015 inches. Hence, we can now *widen* the individual tolerances, which should reduce the cost of manufacturing the parts. This time we will assume

Table 2

		PROPORTIONAL		PRECISION FACTOR	
Part	*Original Tolerance*	*Worst Case*	*Statistical (3σ)*	*Worst Case*	*Statistical (3σ)*
A	0.00150	0.00150	0.00105	0.00150	0.00150
B	0.00800	0.00378	0.01116	0.00312	0.00967
C	0.00250	0.00250	0.00250	0.00250	0.00250
D	0.00200	0.00094	0.00279	0.00115	0.00356
E	0.00600	0.00283	0.00837	0.00308	0.00955
F	0.00200	0.00094	0.00279	0.00115	0.00356
G	0.00250	0.00250	0.00250	0.00250	0.00250
Assembly Tolerance		0.01500	0.01500	0.01500	0.01500
Proportional Factor		0.47222	1.39526	0.00156	0.00484

that the tolerances vary with the cube root of their respective target values, and therefore use the precision factor to allocate the excess tolerance.

$$0.015^2 = .0015^2 + .0025^2 + .0025^2 + P_c^2 \cdot (8.0^{2/3} + .400^{2/3} + 7.711^{2/3} + .400^{2/3})$$

$$P_c = 0.00484$$

Then we obtain the new, wider tolerances,

$$T_B = 0.00484 \cdot (0.008)^{1/3} = 0.00976$$
$$T_D = 0.00484 \cdot (0.002)^{1/3} = 0.00356$$
$$T_E = 0.00484 \cdot (0.006)^{1/3} = 0.00955$$
$$T_F = 0.00484 \cdot (0.002)^{1/3} = 0.00356$$

Note again that we did not use the natural tolerances of the parts when using the constant precision factor allocation. We simply made the assumption that their tolerances were functions of the size of their dimensions.

The above results are shown in Table 2. The worst-case constant precision factor and statistical-proportional scaling methods have also been calculated and included in the table for comparison.

Figure 3 shows graphically the differences among the four methods. Notice how much tighter we force the production tolerances to be when using the worst case method. It's also interesting to see how similar the precision factor method is to the proportional even though it requires no information about the natural tolerances of the individual parts.

LIMITATIONS TO WORST CASE AND STATISTICAL METHODS

There are some limitations to using the worst case and statistical methods for setting tolerances:

- The worst case tolerances tend to be too tight and therefore costly.
- The statistical method tends to estimate higher assembly yields (% w/o defects) than actually occur. Also, it doesn't allow for skewness or bias common in manufacturing situations.

ESTIMATED MEAN SHIFT MODEL

Chase and Greenwood (1987) introduced the estimated mean shift model to remove some of the limitations of the other models and to create a common ground between the traditionally conservative design department and the more liberal manufacturing. We describe it briefly here.

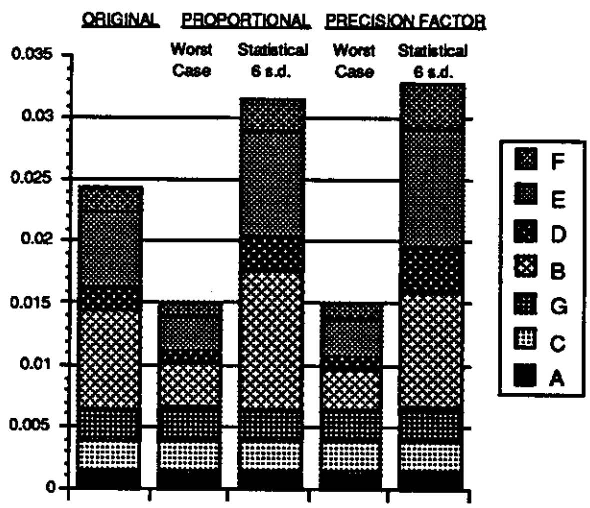

Figure 3. *Results using the four combinations.*

In this model the designer must reach agreement with manufacturing in estimating the possible bias for each component, i.e. the potential shift in the distribution mean. They estimate this by defining a zone about the midpoint of the tolerance range within which the distribution mean might be expected to shift. The zone is specified as a fraction, m, of the natural 6σ range. Figure 4 presents the method graphically. The final assembly tolerance is then calculated using,

$$T_{Assembly} = \Sigma|m_i \cdot T_i| + \left[\Sigma\left((1-m_i)^2 \cdot T_i^2\right)\right]^{1/2}$$

One can see that the equation involves both worst case and statistical components. When $m_i=1$, it reduces to a worst case model, and it becomes statistical at $m_i=0$.

The advantages of the estimated mean shift model are said to be that the engineer:

- can simulate the entire continuum from narrow worst case limits to larger statistical ones, depending on certainty of the part dimensional means;
- has full flexibility to mix both worst case (i.e. parts with high potential for a shift in the mean) and statistically summed parts;
- can be conservative in the design stage and later more liberal by reducing m when manufacturing data become available.

OTHER TOOLS

Several additional tools are available for setting tolerances more accurately than the often crude basic methods. These also tend to be more time consuming and/or expensive. Several of these methods remain rather academic and aren't widely used in industry, but others can be extremely valuable in setting tolerances.

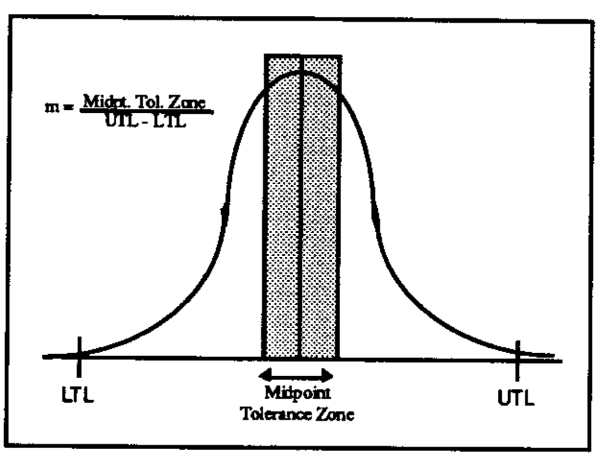

Figure 4. *Estimated mean shift model.*

OPTIMIZATION

As stated above, tolerance analysis involves not only the assurance of expected performance in the final product, but also its manufacture at a low cost. Optimization tolerancing adjusts individual part tolerances to achieve minimum cost of manufacture while maintaining the overall required tolerance. The key element here is the determination of the cost vs. tolerance functions of the individual parts as in Figure 5. These can be obtained crudely by figuring the fixed and variable costs of manufacturing the parts at various tolerance levels. Or if the data are available, it can be done by regressing previous costs on the tolerance levels achieved. This method certainly depends greatly on the accuracy of the manufacturing accounting system.

In any case, once the cost functions for all the individual parts are predicted, we sum them to get the objective function. We then try to minimize this function subject to the constraint that the tolerances must sum (either worst case or statistically, as specified) to within the specification. Any linear programming package could perform this iterative algorithm. Simple problems could be solved by hand

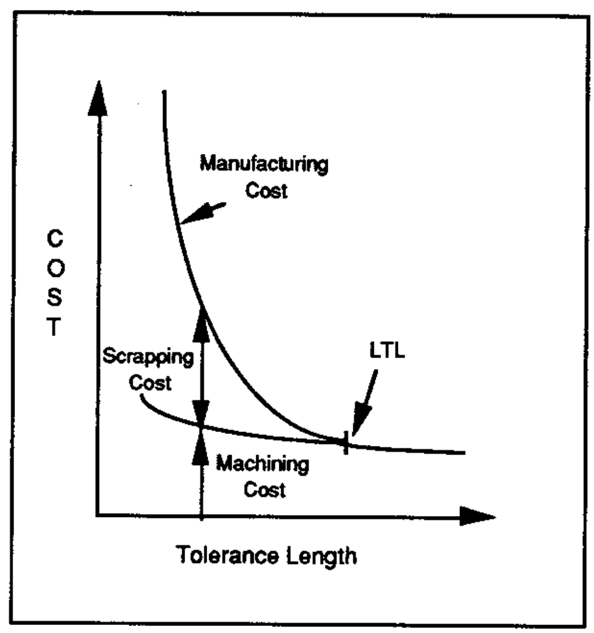

Figure 5. *Example of manufacturing cost vs. tolerance length.*

using the simplex method. Spotts (1973) and Chase and Greenwood (1988) use LaGrange Multipliers to compute the cost-optimal tolerance specifications non-iteratively. See also the Peter's Method (Peters [1970]).

MONTE CARLO SIMULATION

When part dimensions are normally distributed and unbiased, the statistical method of tolerance analysis is arguably the best one to use. However, it is commonly the case in manufacturing that the distributions are skewed and/or biased. In these cases the statistical method is really not valid, but we still want more realistic tolerances than provided by the worst case method. The Monte Carlo Simulation is useful in this situation and is described by Spotts (1983). It essentially generates random numbers from the distributions of the part dimensions and simulates the distribution of their vector sum in the final assembly. The engineer then compares this distribution to the required specifications to estimate the number of defects that would be produced. The engineer can also perform experiments to investigate what happens to the distribution of the final dimension when the midpoints and tolerances of the part dimensions are adjusted. The primary drawback to the Monte Carlo Simulation is that it can take a considerable amount of computer time. Shapiro and Gross (1981) provide a detailed explanation of the Monte Carlo Simulation method.

LIMITATIONS TO USING THESE TOOLS

Some obstacles to the use of these other tools are:

- In the early product design stages the engineers often have little information regarding the distributions of the part dimensions or the costs of producing them.
- The methods are computation intensive and/or complex.

Because of these practical problems, several of the advanced techniques remain academic exercises and are not widely used in industry.

SETTING INITIAL DESIGN TOLERANCES

When designing basic machined parts, electronic devices, or other well-studied products the engineer often has tables to use as guides in setting initial design tolerances. Even if there are no tables the manufacturer often has produced similar products whose data can be used as a guide in setting tolerances on the new design. However, in cases where the engineer is "in the dark" or simply wants confirmatory data, they can use the following tools to obtain a better understanding of the manufacturing variability.

BY SAMPLING

When the engineer has the ability to produce several pilot parts at low cost their measurements can be used to estimate the distribution of the part dimensions. The sample mean $\bar{x}$ and standard deviation s of the pilot set provide estimates of the true parameters μ and σ, respectively. The normality of the data should be checked with a normal probability plot and perhaps additionally with a Chi-square test and an investigation of time order dependency.

However, even if normality can be assumed it would be incorrect to expect that 99.7% of the manufactured products will lie within 3s of $\bar{x}$ since s and $\bar{x}$ are estimates themselves. Instead, we can only assume that a fixed portion, γ, of the intervals $\bar{x} \pm ks$ will include $100(1-\alpha)\%$ or more of the distribution. Then, using nomographs as found in Banks and Heikes (1984) to find k based on the choice of γ and α, we can set the lower tolerance limit at $\bar{x} - ks$ and the upper at $\bar{x} + ks$. Refer also to Banks (1989) for how to perform this function when the underlying distribution cannot be assumed normal.

SMALL SAMPLING

More typically, the engineer doesn't have the resources to manufacture a large number of products in a pilot run. Instead, they are only allowed a handful of development tests from which to make an estimate of the tolerance. According to Juran (1988), the data could be plotted on normal probability paper and then extrapolated to the extreme ends of the distribution. This provides crude but safer tolerances than would be obtained by a simple glance at the data.

EXPERIMENTATION

Juran (1988) describes an example of setting tolerances on a thermostat. The thermostat is required to turn on and off at specified temperatures. The engineer designs a development test in which the on/off temperatures are varied systematically and the resulting performance characteristic is measured. In this example the engineer may want to study how the range between the start up and shut off temperatures affects the temperature variation in a given room. A scatter plot (as in Figure 6) can then be constructed and a regression equation fit to the data. This equation is then a simple tool for understanding the tolerances required on the temperature range settings to obtain the desired temperature consistency.

LOSS FUNCTION

Taguchi (1986) defines the loss function as a function of the cost associated with the deviation of a dimension from its target. This method of setting tolerances is based on the overall 'cost to society' rather than solely on the cost to the manufacturer. Dr. Taguchi assumes a parabolic shape to the loss function as shown in Figure 7, i.e. the function is of the form,

$$L(y) = k \cdot (y - t)^2$$

where $L(y)$ is the added cost, y is the performance characteristic, t is the target value, and k is some constant. Now let A_o be the average loss incurred when the product is returned by the customer due to being out of tolerance, and s_o be the specification half-width.

The cost A_o includes machining or replacing the part as well as the transportation to the manufacturer and back. Once this is estimated we can find the constant k from the equation

$$L(y) = A_o = k \cdot ((t - s_o) - t)^2.$$

From this we get

$$k = \frac{A_o}{s_o^2}$$

and hence

$$L(y) = \frac{A_o}{s_o^2}(y - t)^2.$$

Let A be the average cost to the manufacturer when a part is inspected in-house and found to be outside the manufacturing tolerance. This could again be the cost of additional machining or of scrapping the part. Substituting these values into the loss function allows us to solve for s,

$$A = \frac{A_o}{s_o^2}(s^2)$$

so that,

$$s = \left[\frac{A}{A_o}\right]^{1/2} \cdot (s_o).$$

becomes our manufacturing tolerance half-width. Dr. Taguchi suggests this method for setting tolerances since it takes into account not only the cost of being outside the manufacturing tolerance, but also that of missing the customer's requirements. Some criticisms of the Loss Function include:

- The function is highly sensitive to the accuracy

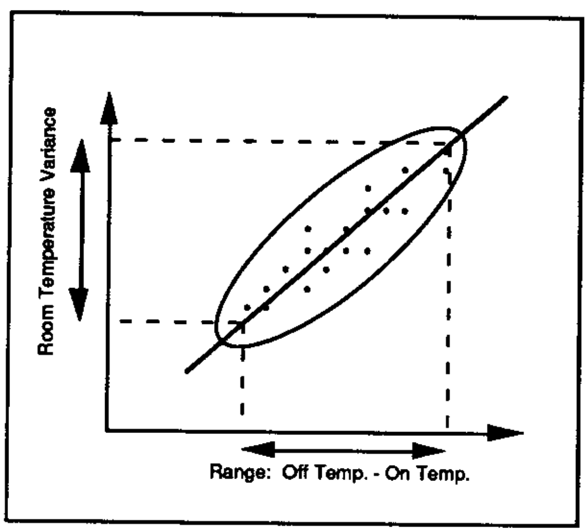

Figure 6. *Performance characteristic vs. tolerance.*

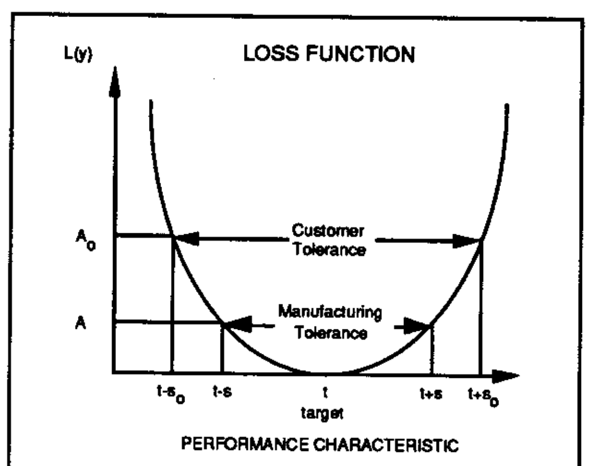

Figure 7. *The loss function.*

Table 3

Operation Number (A)	Nominal diameter at completion of operation (B)	Estimated natural tolerance range (6sd) of increase in diameter due to operation (C)	Specified tolerance range of diameter at completion of operation (D)	$D^2 - C^2$ (E)	Computed tolerance range before start of operation $E^{1/2}$ (F)	Computed specification at start of operation (G)
8	0.750	0.016	0.050	0.002244	0.047	0.710±0.023
7	0.710	0.021	0.047	0.001803	0.042	0.640±0.021
6	0.640	0.018	0.042	0.001479	0.038	0.585±0.019
5	0.585	0.007	0.038	0.001430	0.038	0.550±0.019
4	0.550	0.013	0.038	0.001261	0.036	0.500±0.018
3	0.500	0.028	0.036	0.000477	0.022	0.300±0.011
2	0.300	0.016	0.022	0.000221	0.015	0.100±0.007
1	0.100	0.008	0.015	0.000157		

of estimating the true costs to both the manufacturer and customer.

- If the product is 100% inspected by either the manufacturer or customer then the loss might be constant outside those tolerance limits.
- The assumption that the function is parabolic might be inappropriate in some cases.

EXAMPLE OF SETTING TOLERANCES AT INTERMEDIATE STAGES

The following example is taken from Grant and Leavenworth (1988). It involves the setting of tolerances at individual stages in the manufacture of electric cable. The process requires several stages, each of which increases the cable's total diameter. The concern is to set the tolerances at the intermediate stages such that the specified final tolerance is met while none of the individual processes are required to have tighter tolerances than what is manufacturable.

The specified final diameter of the electric cable is 0.75 and from control charts it is estimated to have a natural tolerance (6σ) of 0.050. These numbers are shown in columns B and D of Table 3. The diameter added to the cable in each of the other seven operations and the associated natural tolerances (based on the capability of each operation) are also shown. From this data we wish to set a tolerance on each of the individual operations. We do this based on the statistical equations,

$$\sigma_{operation(i)} = \sqrt{\sigma^2_{sum} - \sigma^2_{operation(i+1)}} \,.$$

In our case,

$$\sigma_{operation(7)} = \sqrt{0.050^2 - 0.016^2} = 0.047,$$

$$\sigma_{operation(6)} = \sqrt{0.047^2 - 0.021^2} = 0.042,$$

$$\vdots$$

etc.

Calculating the remaining values leaves operation 1 with an available tolerance of 0.015. However, its 6σ natural tolerance is only 0.008, which means our calculation would allow for a much wider tolerance on operation 1 than is necessary. The engineer could then return to column C and increase each of the operations' tolerance ranges either by a fixed amount or proportional to the tolerance range specified, or perhaps proportional to the difficulty of maintaining the tolerances at each operation. The appropriate choice depends on the situation. These trial-and-error type recalculations can be quickly performed using spreadsheet software.

EXAMPLE OF MATING SOCKET SET PARTS

The second example taken from Grant and Leavenworth (1988) is an illustration of the relationship among chosen nominal dimensions and

Table 4

	MEAN	STANDARD DEVIATION
Male Attachment	.5005	.0015
Female Socket	.5120	.0010
Clearance	.0115	.0018

tolerances, based on their fits and clearances. The critical dimensions are the width of the male square attachment and that of the female square socket. These are shown in Table 4. The minimum desired clearance between the mating dimensions is stated to be 0.004 inches, but it should not exceed 0.015 inches, as this will result in a fit that is too loose.

Figure 8a shows the initial situation. The manufacturer produces essentially no combinations that don't fit together, i.e. the smallest female sockets are large enough to accommodate the largest male attachments. The distribution of their resulting clearances is shown in Figure 8b and is based on the statistical sum of the two normal distributions. It can be seen that a certain percentage of the attachment-socket combinations are too loose using the current target values.

Hence, we might want to move the target dimensions closer together, as in Figure 8c, to obtain a tighter fit. The question then is how far should they be shifted? The answer is based on another question. What risk does the manufacturer want to take that, somewhere, sometime, a mechanic will pick up a socket and extension with less than .004 clearance between them? By simply taking advantage of the properties of the normal distribution, the engineer can estimate the probability that the parts will fit too tightly or too loosely. To review, the probabilities can be determined using the standard normal probability table and the equations

$$P(Loose) = P(MEAN_{Clearance} > 0.015) = P(Z = \frac{0.015 - MEAN_{Clearance}}{\sigma_{Clearance}}) = 1 - \Phi(\frac{0.015 - MEAN_{Clearance}}{\sigma_{Clearance}}),$$

$$P(Tight) = P(MEAN_{Clearance} < 0.004) = P(Z = \frac{0.004 - MEAN_{Clearance}}{\sigma_{Clearance}}) = 1 - \Phi(\frac{0.004 - MEAN_{Clearance}}{\sigma_{Clearance}})$$

where

$$MEAN_{Clearance} = MEAN_{Socket} - MEAN_{Attachment},$$

and

$$\sigma_{Clearance} = \sqrt{\sigma^2_{Socket} + \sigma^2_{Attachment}}\,.$$

If we choose several mean clearances and plug those values into the equations we can generate something similar to Table 5. This listing of possible nominal clearances and the associated probabilities of producing combinations that are too tight/loose allows all concerned (design, manufacturing, accounting, etc.) to discuss and choose an alternative.

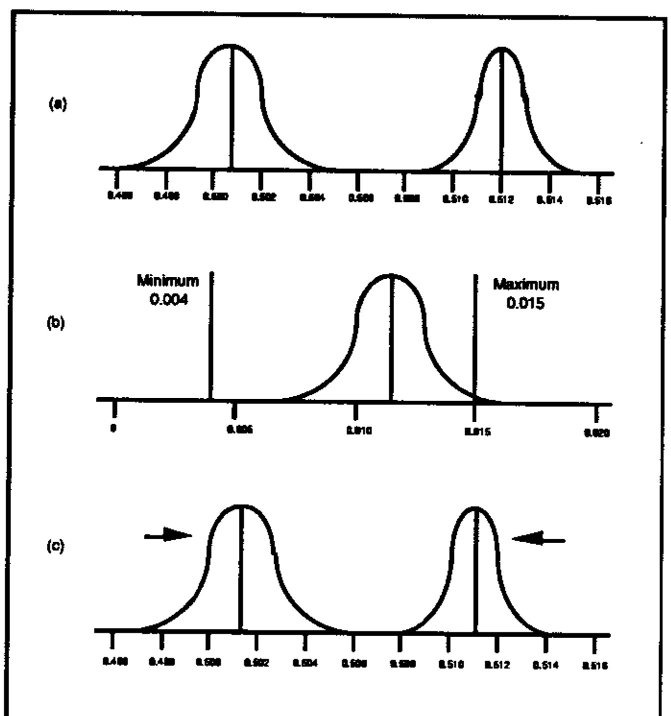

Figure 8. *Adjusting male/female socket fit.*

Table 5

Difference in inches	Probability of clearance smaller than 0.004 in.	Probability of clearance larger than 0.015 in.
0.0115	0.0000	0.0262
0.0110	0.0000	0.0132
0.0105	0.0002	0.0062
0.0100	0.0004	0.0027
0.0095	0.0011	0.0011
0.0090	0.0027	0.0004
0.0085	0.0062	0.0002
0.0080	0.0132	0.0001
0.0075	0.0262	0.0000

SUMMARY

In order to compete successfully in today's market, products must be developed quickly and manufactured with a high level of quality. Tolerance analysis and allocation are important activities in designing products quickly and successfully and with less manufacturing cost. Basing tolerance settings on scientific methods and statistical principles can improve reliability of information and hence decision making early in the development process. Statistical case analysis and proportional or constant precision factor allocation should be effective in most manufacturing situations. They have wide-ranging relevance given the importance of the normal distribution and are relatively easy to calculate. In addition, organizations that are further along in using data-driven decision making might find the estimated mean shift model to be very effective for improving the communication and teamwork between design and manufacturing.

Given the increased complexity in today's manufactured products and advances in computing power, we might expect Monte Carlo Simulation to become more widely used in industry. Dimensions are often not normally distributed (e.g. electronics), and the Monte Carlo Simulation can be useful for summing these more complex tolerances.

It is important for engineers to understand the Loss Function intuitively. We should be thinking not only about manufacturing costs, but also in terms of the cost to the consumers and even to society as a whole. However, it might be difficult to use successfully in practice. It doesn't take into account the process capability and is very dependent on the accuracy of accounting systems.

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