A Note on the Effect of Skewness, Kurtosis, and Shifting on One-Sample T and Sign Tests

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Abstract

We extend previous studies of the effects of skewness, kurtosis, and shifting of the location parameter on the size and power of the t and sign tests for the one-sample case. The sign test is often recomended for skewed populations, however simulations show that the power of the t-test exceeds that of the sign test when the shift is in the opposite direction of the skewness in the parent population. Furthermore, our simulations demonstrate that the power of the sign test is diminished as kurtosis in the parent population decreases.

1 Introduction

The one-sample Student's t-test and the sign test are presented in most elementary statistics textbooks. The sign test is offered as the alternative to the t-test when the normality assumption is violated. To examine the effect of departures from normality on the size and power of the t-test Rhiel and Chaffin (1996) studied the power for the case of skewness in the direction of the rejection region tail as well as the observed level of signicicance for both left- and right-tailed t-tests under right skewed parent populations. Chaffin and Rhiel (1993) have demonstrated that the observed significance level (size) of the one-sample t-test is higher than the nominal level if the skewness of the parent population is in the opposite direction of the rejection region tail and is lower than nominal when the skewness of the parent population is in the same direction as the rejection region tail. We show the differences in power for shifts to both the left and right and highlight how the power is affected when the parent population is skewed. Moreover, we examine the effects of skewness, kurtosis, shifting, and the direction of the test on the power of the sign test as well.

Much has been written on the fact that the distribution of the t-statistic inherits a skewness that is opposite of the parent population. Boos and Hughes-Oliver (2000) clearly illustrate this fact and show that t-intervals constructed for right-skewed parent populations, for example, will have upper and lower bounds that are to the left of what they would be under a normally distributed parent, and vice-versa. In the context of hypothesis testing this translates into true tail probabilities that are higher than nominal in the upper tail, resulting in fewer rejections, and lower than nominal tail probabilities in the lower tail, resulting in more rejections than there would be under a normal parent for one-tailed tests.

While it is true that the sign test requires no assumptions regarding the distribution of the parent population, skewness, shifting, and particularly kurtosis affect the power of the test. The power of the sign test is greatly diminished as kurtosis decreases. Simulations by Ott and Longknecker (2001) show that the t-test is uniformly more powerful than the sign test for normal parent populations and that the sign test is more powerful in cases of heavily tailed or highly skewed parent populations. Our study agrees for skewed parent populations when the shift is in the same direction as the skewness, but not when the shift is in the opposite direction. Simulations by Randles and Wolfe (1979) showed that the power of the sign test is lower for low kurtosis distributions than it is for high kurtosis distributions. We also demonstrate the vulnerability of the sign test to low kurtosis parent populations and provide insight as to why this is so.

2 Methodology

Computer simulations with a broad set of of parent populations were performed using Matlab (Mathworks (2000)). We selected three skewed distributions from the chi-squared family with varying degrees of skewness (2, 4, and 8 degrees of freedom), three low kurtosis distributions matching those used by Chaffin and Rhiel (1993), and three high kurtosis distributions from the contaminated normal family (kurtosis 6, 9, and 12). Simulation results were also obtained for a normal parent population for the sake of comparison. Monte Carlo simulations of size 250,000 were run for each distribution using sample sizes $n = 10(10)100(20)200$ and various significance levels. Moreover, in order to make valid comparisons between the t and sign tests, we used levels of significance determined by sample size and the binomial table that were near 0.01, 0.025, 0.05, and 0.10 for both one- and two-tailed tests in the case of a skewed parent population and two-tailed tests for symetric parent populations. Power was computed for shifts of 0.2σ to 2σ in increments of 0.2σ, where σ is the population standard deviation, both to the left and to the right. The simulation errors for levels of significance .05 and 0.10 are 0.0004 and 0.0006, respectively, and the largest sampling error, associated with power 0.50, is 0.001.

3 Results

It is well known that under the null hypothesis the sampling distribution of the t-statistic $t_0 = \sqrt{n}(\bar{x} - \mu_0)/s$ is negatively skewed when the parent population is positively skewed, and vice versa. Figure 1 shows the relationship between the skewness coefficient of the t-statistic and the size of the shift for the $\chi^2(2)$ parent population and samples of size 20. Monte Carlo simulations of size 1,000,000 for shifts from -1σ to 1.5σ in increments of 0.1σ show that a shift to the left causes even more negative skewness in t_0 while a shift to the right will increase the skewness coefficient rather quickly even to the point of positive skewness. The direction of the shift is certain to be important when the parent population is skewed. In the case of right skewness, as with $\chi^2(2)$, the distribution of the t-statistic will be skewed to the right and the observed level of significance (size) of the test is higher than the nominal level for left-tailed tests and lower for right-tailed tests. Figure 2 illustrates the effect of the direction of the test on the power of the t-test in the case where there is a shift of size 0.5σ. It can be seen in Figure 2 that a shift to the right results in a higher skewness coefficient, which results in higher power while a shift to the left decreases the skewness coefficient and, hence, a decline in power for the t-test. The skewness coefficients for the shifts to the left and right in Figure 2 are approximately -1.25 and 0, respectively.

We found that two-tailed tests under positively skewed parents tend to have observed significance levels that are slightly higher than nominal because the discrepancy for the left-tailed test slightly exceeds the discrepancy

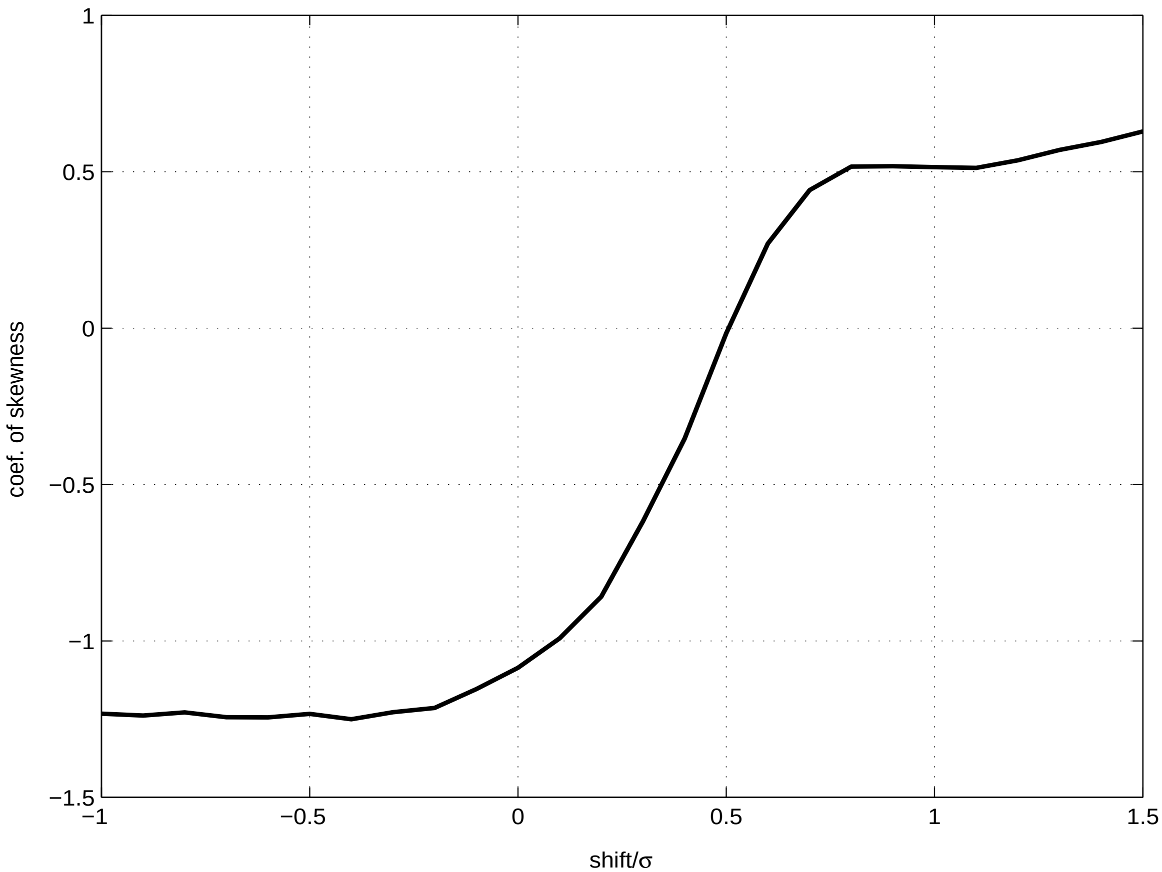

Figure 1: The effect of the size and direction of the shift on skewness of the t-statistic when the parent population is $\chi^2(2)$ for $n = 20$.

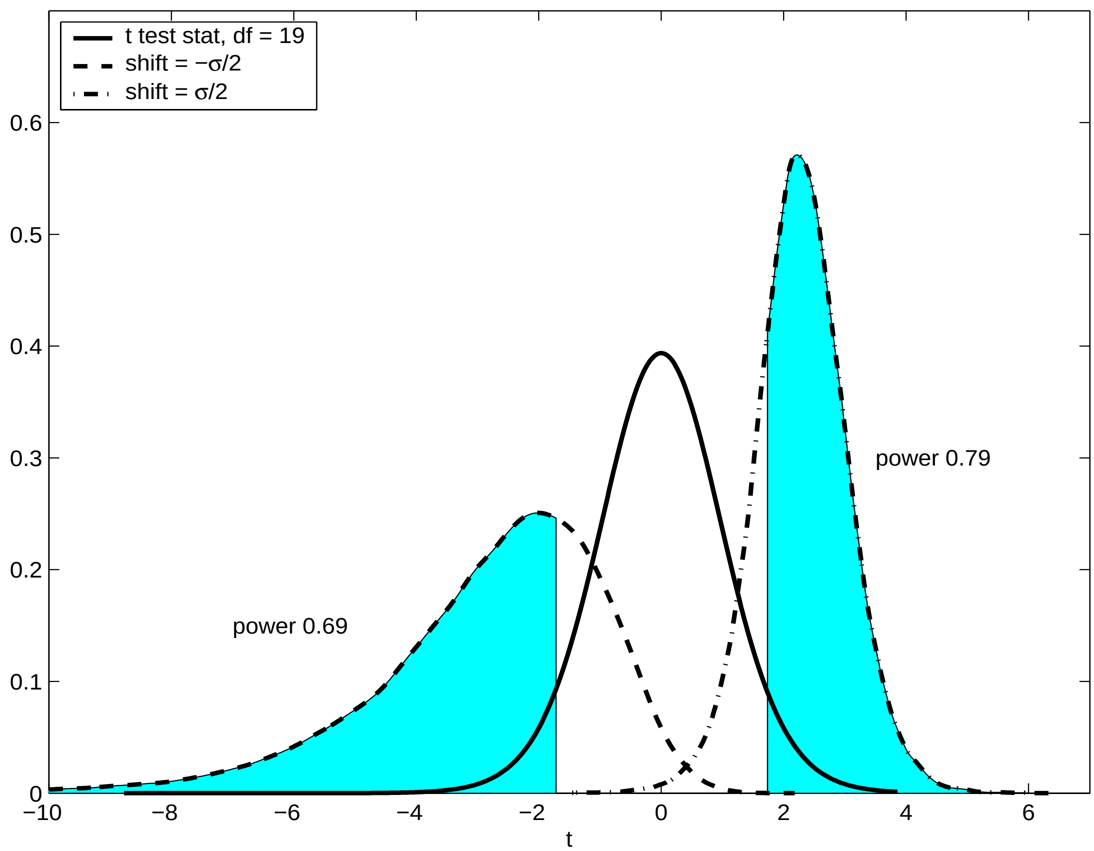

Figure 2: Power of left- and right-tailed t-tests for shifts of $\pm 0.5\sigma$ under a $\chi^2(2)$ parent population for $n = 20$ and $\alpha = 0.05$.

for the right-tailed test. When there is a shift in the underlying parent population, the power of the t-test is not uniformly higher or lower for the skewed parent than for the normal parent population. A comparison of the power of the two-tailed t-test for normal and $\chi^2(2)$ parent populations for samples of size 20 and $\alpha = 0.05$ is plotted in Figure 3. For shifts to the right less than 0.5σ, the power for the test when the parent is normal is higher than when the parent is $\chi^2(2)$, but beyond shifts of 0.5σ the opposite is true. However, for shifts to the left the power for the $\chi^2(2)$ parent is higher until the shift is less than roughly -0.6σ.

Tables 1 and 2 show that for the right-skewed distribution $\chi^2(2)$, the observed level of significance is always higher than the nominal level for left-tailed tests (*i.e.* when the direction of the test is opposite that of the skewness of the parent population). Conversely, the observed significance level is always lower than nominal when the direction of the test and the skewness of the parent population are the same. This is a result of the skewness in the sampling distribution of the t-statistic that occurs when the parent population is skewed. The relationship between the power of the t-test for normal parent populations and the power of the t-test for a $\chi^2(2)$ parent population for $n = 20$ and $\alpha = 0.05$ shown in Figure 3 is also evident for all of the sample sizes displayed in Tables 1 and 2 for $\alpha = 0.05$ and $\alpha = 0.10$. Specifically, the power of the right-tailed test starts out lower for the $\chi^2(2)$ parent than for the normal parent but then overtakes it as the shift increases. The larger the sample size, the smaller the shift required for the crossover to occur. The power of the left-tailed test starts out higher for the $\chi^2(2)$ parent than for the normal parent but then declines below it as the shift increases. Again, the larger the sample size, the smaller the shift required for the crossover to occur.

While there are no distributional assumptions necessary for the sign test, our simulations show that the power of the sign test is affected by skewness and kurtosis in the parent population. In the case of a skewed parent population, such as $\chi^2(2)$, the power of the sign test uniformly exceeds that of the t-test as long as the shift and direction of the test are the same as the direction of skewness of the parent population. However, Figure 4 and Table 3 show that the t-test has superior power when the shift and direction of the test are in the opposite direction of the skewness of the parent population.

Figure 5 shows the large variation in the power of the sign test for three symmetric distributions with what we might consider low ($k = 1.8$), medium ($k = 3$), and high($k = 9$) kurtosis. The power of the sign test is particularly

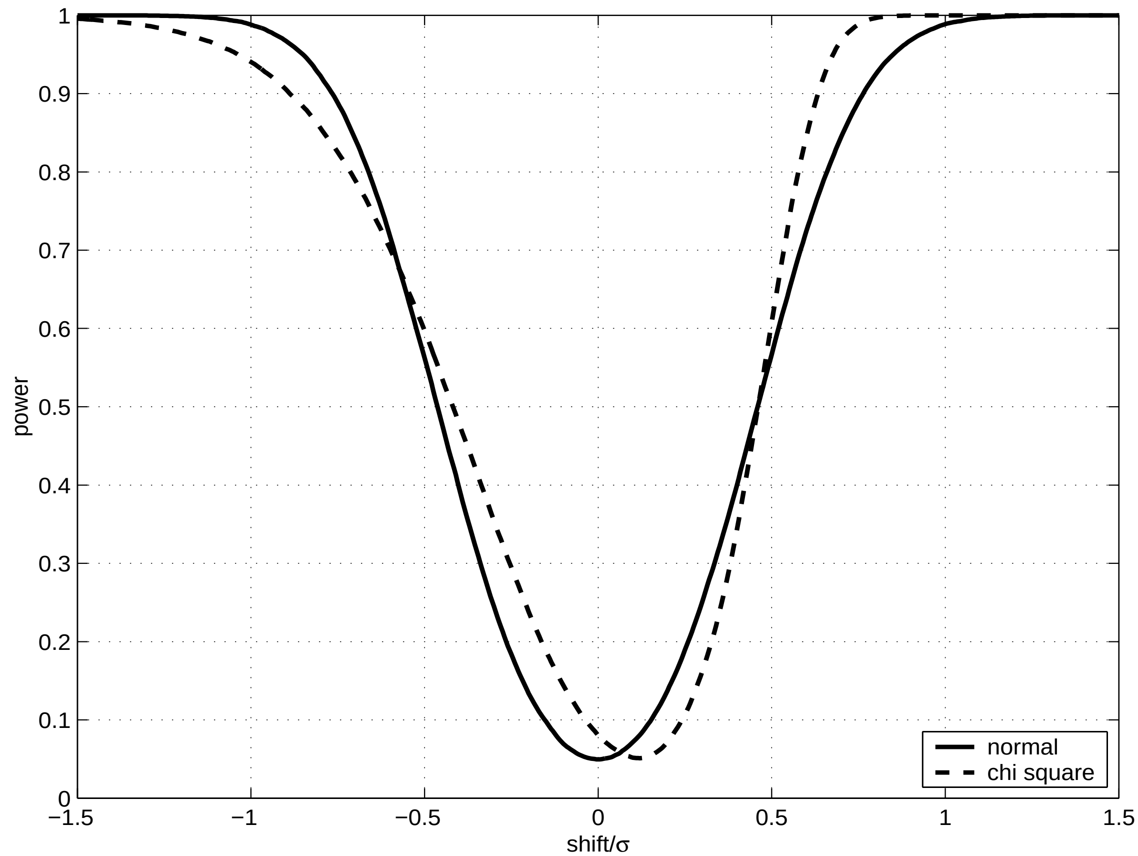

Figure 3: The power of the t-test is not uniformly higher or lower for the $\chi^2(2)$ parent than for the normal parent population for $n = 20$ and $\alpha = 0.05$.

$\alpha = 0.05$		Normal		$\chi^2(2)$		Uniform		CN $k = 9$	
n	shift/σ	Left	Right	Left	Right	Left	Right	Left	Right
	−0.75	0.707	0.000	0.698	0.000	0.689	0.000	0.672	0.000
	−0.50	0.427	0.001	0.517	0.000	0.395	0.002	0.448	0.000
	−0.25	0.181	0.009	0.310	0.002	0.165	0.011	0.200	0.002
10	0.00	0.050	0.050	0.132	0.014	0.050	0.050	0.041	0.042
	0.25	0.009	0.180	0.031	0.090	0.011	0.165	0.003	0.201
	0.50	0.001	0.428	0.002	0.389	0.002	0.396	0.000	0.449
	0.75	0.000	0.707	0.000	0.869	0.000	0.690	0.000	0.674
	−0.75	0.944	0.000	0.882	0.000	0.951	0.000	0.916	0.000
	−0.50	0.694	0.000	0.684	0.000	0.687	0.000	0.694	0.000
	−0.25	0.284	0.003	0.378	0.001	0.274	0.004	0.315	0.001
20	0.00	0.049	0.051	0.109	0.019	0.050	0.050	0.048	0.048
	0.25	0.003	0.287	0.010	0.212	0.004	0.273	0.001	0.316
	0.50	0.000	0.697	0.000	0.788	0.000	0.685	0.000	0.696
	0.75	0.000	0.944	0.000	0.998	0.000	0.951	0.000	0.917
	−0.75	0.991	0.000	0.957	0.000	0.994	0.000	0.983	0.000
	−0.50	0.847	0.000	0.801	0.000	0.850	0.000	0.843	0.000
	−0.25	0.379	0.001	0.443	0.000	0.368	0.002	0.412	0.000
30	0.00	0.050	0.051	0.098	0.023	0.049	0.049	0.049	0.050
	0.25	0.001	0.379	0.004	0.333	0.002	0.369	0.000	0.410
	0.50	0.000	0.849	0.000	0.940	0.000	0.850	0.000	0.842
	0.75	0.000	0.991	0.000	1.000	0.000	0.993	0.000	0.983
	−0.75	1.000	0.000	0.998	0.000	1.000	0.000	1.000	0.000
	−0.50	0.986	0.000	0.957	0.000	0.988	0.000	0.980	0.000
	−0.25	0.608	0.000	0.614	0.000	0.601	0.000	0.616	0.000
60	0.00	0.050	0.050	0.082	0.030	0.051	0.048	0.050	0.050
	0.25	0.000	0.605	0.000	0.622	0.000	0.605	0.000	0.616
	0.50	0.000	0.986	0.000	0.999	0.000	0.986	0.000	0.980
	0.75	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
	−0.75	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
	−0.50	1.000	0.000	0.995	0.000	1.000	0.000	0.999	0.000
	−0.25	0.799	0.000	0.776	0.000	0.799	0.000	0.790	0.000
100	0.00	0.050	0.050	0.073	0.033	0.050	0.048	0.050	0.050
	0.25	0.000	0.798	0.000	0.844	0.000	0.800	0.000	0.791
	0.50	0.000	1.000	0.000	1.000	0.000	0.999	0.000	0.999
	0.75	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000

Table 1: Power of the t-test for various shifts and sample sizes under normal, $\chi^2(2)$, uniform, and contaminated normal parent populations, $\alpha = 0.05$.

$\alpha = 0.10$		Normal		$\chi^2(2)$		Uniform		CN $k = 9$	
n	shift/σ	Left	Right	Left	Right	Left	Right	Left	Right
	−0.75	0.836	0.000	0.791	0.000	0.835	0.000	0.785	0.000
	−0.50	0.590	0.003	0.624	0.001	0.565	0.004	0.584	0.000
	−0.25	0.300	0.021	0.404	0.008	0.280	0.023	0.324	0.013
10	0.00	0.100	0.099	0.188	0.048	0.097	0.097	0.104	0.104
	0.25	0.021	0.298	0.049	0.220	0.023	0.281	0.013	0.327
	0.50	0.003	0.589	0.004	0.638	0.004	0.567	0.000	0.585
	0.75	0.000	0.836	0.000	0.974	0.000	0.834	0.000	0.787
	−0.75	0.977	0.000	0.934	0.000	0.981	0.000	0.960	0.000
	−0.50	0.817	0.000	0.781	0.000	0.816	0.000	0.803	0.000
	−0.25	0.424	0.009	0.485	0.004	0.415	0.010	0.449	0.005
20	0.00	0.099	0.100	0.164	0.058	0.099	0.099	0.105	0.105
	0.25	0.008	0.426	0.018	0.389	0.009	0.413	0.005	0.450
	0.50	0.000	0.818	0.000	0.911	0.000	0.815	0.000	0.804
	0.75	0.000	0.977	0.000	1.000	0.000	0.982	0.000	0.960
	−0.75	0.997	0.000	0.980	0.000	0.998	0.000	0.994	0.000
	−0.50	0.921	0.000	0.876	0.000	0.925	0.000	0.913	0.000
	−0.25	0.527	0.004	0.558	0.002	0.520	0.004	0.550	0.002
30	0.00	0.100	0.101	0.153	0.065	0.098	0.098	0.105	0.104
	0.25	0.004	0.528	0.008	0.522	0.005	0.520	0.002	0.548
	0.50	0.000	0.922	0.000	0.980	0.000	0.924	0.000	0.913
	0.75	0.000	0.997	0.000	1.000	0.000	0.998	0.000	0.994
	−0.75	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
	−0.50	0.995	0.000	0.979	0.000	0.996	0.000	0.992	0.000
	−0.25	0.741	0.001	0.726	0.001	0.736	0.001	0.742	0.000
60	0.00	0.100	0.099	0.135	0.074	0.100	0.098	0.103	0.102
	0.25	0.001	0.739	0.001	0.776	0.001	0.739	0.000	0.743
	0.50	0.000	0.995	0.000	1.000	0.000	0.995	0.000	0.992
	0.75	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
	−0.75	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
	−0.50	1.000	0.000	0.998	0.000	1.000	0.000	1.000	0.000
	−0.25	0.887	0.000	0.861	0.000	0.888	0.000	0.879	0.000
100	0.00	0.100	0.099	0.126	0.078	0.100	0.098	0.101	0.101
	0.25	0.000	0.886	0.000	0.925	0.000	0.887	0.000	0.879
	0.50	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
	0.75	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000

Table 2: Power of the t-test for various shifts and sample sizes under normal, $\chi^2(2)$, uniform, and contaminated normal parent populations, $\alpha = 0.10$.

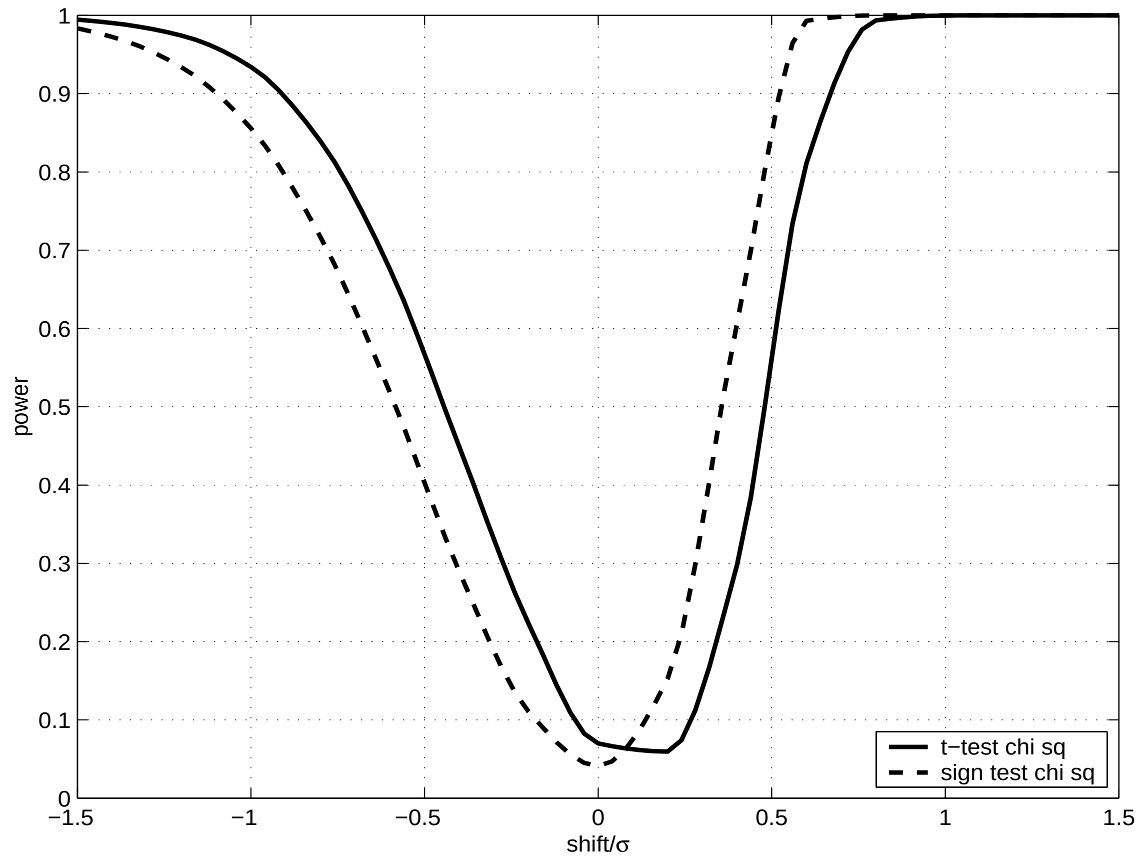

Figure 4: Power comparison for the sign test versus400.00 the t-test when the parent population is $\chi^2(2)$ for $n = 20$ and $\alpha = 0.0414$.

sensitive to kurtosis for relatively small shifts, even among large sample sizes. It is evident that the power of the sign test for the high kurtosis parent population is uniformly larger than the power of the tests for the medium and low kurtosis parent populaitons. Likewise, the power of the sign test under the medium kurtosis parent is uniformly higher than the power for the low kutrosis parent population. In other words, in the range of kurtosis from 1.8 to 9 the power of the sign test increases as kurtosis increases.

The reason for this is that samples from low kurtosis populations have fewer observations in the center and more observations between, say, 1 and 2 standard deviations from the center than samples from high kurtosis populations. As a result, small shifts do not result in very many observations "moved" to the other side of the median, so the sign test is not as likely to reject the null hypothesis, resulting in lower power. This idea can be taken to an extreme by considering a sample containing an equal number of two distinct values, say -1 and 1, for example. Such a population will have kurtosis equal to 1, the smallest value possible. In this case, any shift less than 1 will result in the power near the level of significance of the test, whereas any shift greater than 1 will result in nearly 100% power. On the other hand, samples from high kurtosis populations have a higher concentration of observations near the center of the distribution by comparison, so even a small shift will result in a lot of observations being "moved" to the other side of the median, leading to more rejections of the null hypothesis and, hence, greater power. In Table 4 the power of ont-tailed $t-$ and sign tests are given for the low, medium and high kurtosis distributions. For levels of significance near 0.05, the observed significance level of each test is not affected by the choice of parent population, except for the t-test with a high kurtosis parent at sample size 10, where it is slightly diminished. For the low kurtosis parent population, the power of the t-test is uniformly higher than that of the sign test for each sample size and shift tested. While simulations by Ott and Longknecker (2001) and Randles and Wolfe (1979) show the superioriy of the power of the sign test to the t-test for a symetric high kurtosis parent population, our simulation demonstrates that the power of the sign test is greatly reduced for a symmetric low kurtosis parent population. That the t-test has higher power than the sign test under a normal parent population may have less to do with the fact that the t-test assumes normality and more to do with the kurtosis of the parent population.

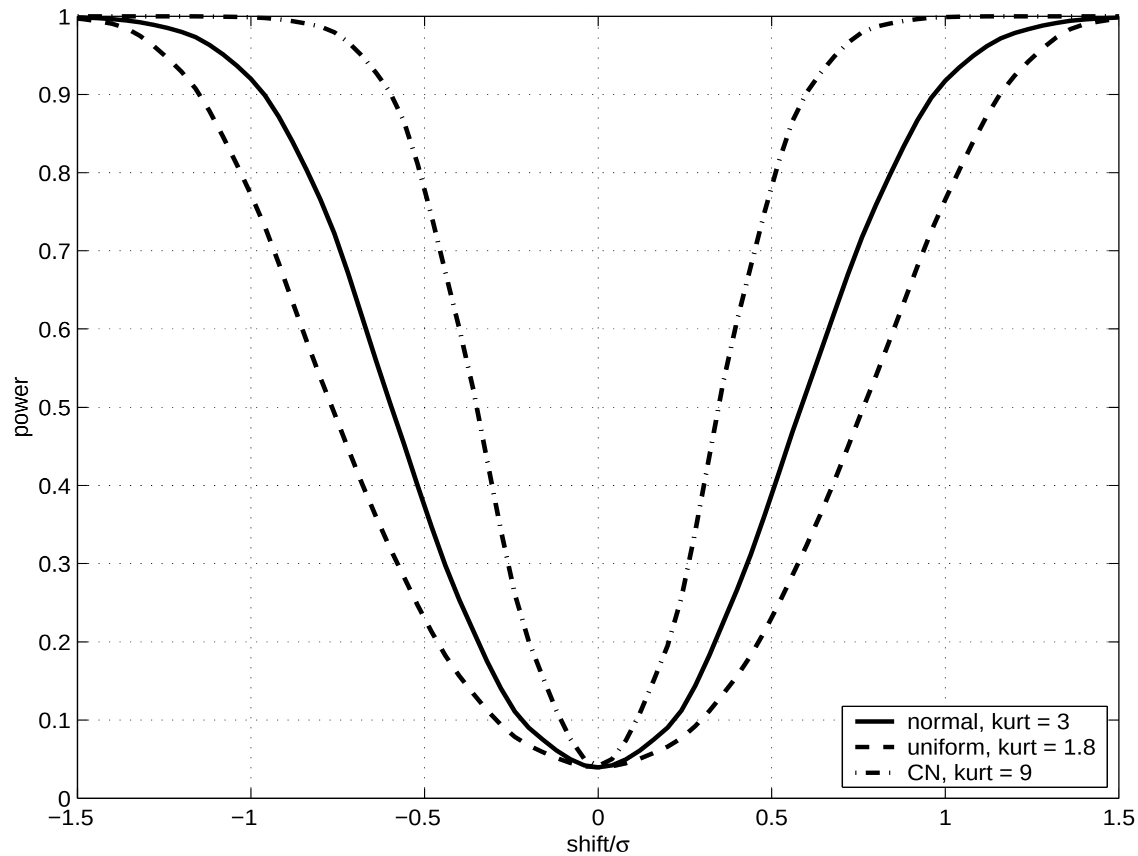

Figure 5: The power of the sign test for low, medium, and high kurtosis parent populations when $n = 20$ and $\alpha = 0.0414$.

	$\alpha \approx 0.05$		t-test				sign test			
			Normal		$\chi^2(2)$		Normal		$\chi^2(2)$	
n	α	shift/σ	Left	Right	Left	Right	Left	Right	Left	Right
		−0.75	0.724	0.000	0.713	0.000	0.595	0.000	0.570	0.000
		−0.50	0.446	0.001	0.534	0.000	0.359	0.002	0.376	0.002
		−0.25	0.192	0.010	0.324	0.002	0.165	0.012	0.186	0.010
10	0.055	0.00	0.055	0.055	0.139	0.016	0.055	0.055	0.055	0.054
		0.25	0.010	0.194	0.033	0.100	0.013	0.166	0.006	0.244
		0.50	0.001	0.447	0.003	0.417	0.002	0.361	0.000	0.749
		0.75	0.000	0.725	0.000	0.889	0.000	0.597	0.000	1.000
		−0.75	0.952	0.000	0.893	0.000	0.853	0.000	0.827	0.000
		−0.50	0.722	0.000	0.704	0.000	0.576	0.000	0.596	0.000
		−0.25	0.311	0.004	0.397	0.001	0.246	0.007	0.282	0.005
20	0.058	0.00	0.058	0.058	0.119	0.024	0.058	0.057	0.058	0.058
		0.25	0.004	0.311	0.011	0.244	0.007	0.247	0.002	0.388
		0.50	0.000	0.722	0.000	0.818	0.000	0.575	0.000	0.952
		0.75	0.000	0.952	0.000	0.999	0.000	0.852	0.000	1.000
		−0.75	0.991	0.000	0.958	0.000	0.941	0.000	0.924	0.000
		−0.50	0.847	0.000	0.801	0.000	0.696	0.000	0.717	0.000
		−0.25	0.376	0.001	0.441	0.000	0.286	0.003	0.333	0.002
30	0.049	0.00	0.049	0.049	0.097	0.022	0.049	0.049	0.050	0.049
		0.25	0.001	0.377	0.004	0.330	0.003	0.287	0.001	0.470
		0.50	0.000	0.848	0.000	0.938	0.000	0.696	0.000	0.990
		0.75	0.000	0.991	0.000	1.000	0.000	0.942	0.000	1.000
		−0.75	1.000	0.000	0.998	0.000	0.998	0.000	0.996	0.000
		−0.50	0.984	0.000	0.954	0.000	0.916	0.000	0.927	0.000
		−0.25	0.590	0.000	0.601	0.000	0.441	0.001	0.519	0.000
60	0.046	0.00	0.046	0.047	0.075	0.027	0.047	0.046	0.045	0.046
		0.25	0.000	0.591	0.000	0.605	0.001	0.444	0.000	0.711
		0.50	0.000	0.984	0.000	0.999	0.000	0.916	0.000	1.000
		0.75	0.000	1.000	0.000	1.000	0.000	0.998	0.000	1.000
		−0.75	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
		−0.50	0.999	0.000	0.995	0.000	0.988	0.000	0.990	0.000
		−0.25	0.783	0.000	0.759	0.000	0.613	0.000	0.705	0.000
100	0.044	0.00	0.044	0.044	0.067	0.029	0.044	0.044	0.045	0.044
		0.25	0.000	0.782	0.000	0.826	0.000	0.613	0.000	0.882
		0.50	0.000	1.000	0.000	1.000	0.000	0.988	0.000	1.000
		0.75	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000

Table 3: Power of t and sign tests for normal and $\chi^2(2)$ parent population.

			t-test			sign test		
n	α	shift/σ	Unif.	Norm.	CN	Unif.	Norm.	CN
10	0.055	0.000	0.055	0.055	0.048	0.055	0.055	0.055
		0.250	0.095	0.102	0.110	0.073	0.089	0.139
		0.500	0.208	0.224	0.233	0.127	0.181	0.309
		0.750	0.354	0.363	0.344	0.214	0.299	0.427
20	0.058	0.000	0.058	0.058	0.057	0.058	0.058	0.057
		0.250	0.151	0.158	0.172	0.096	0.127	0.235
		0.500	0.357	0.361	0.359	0.200	0.288	0.453
		0.750	0.479	0.476	0.463	0.335	0.426	0.497
30	0.049	0.000	0.050	0.049	0.048	0.050	0.049	0.049
		0.250	0.185	0.189	0.205	0.102	0.145	0.293
		0.500	0.424	0.424	0.421	0.241	0.348	0.488
		0.750	0.497	0.495	0.491	0.396	0.471	0.500
60	0.046	0.000	0.046	0.047	0.047	0.046	0.047	0.046
		0.250	0.296	0.296	0.300	0.146	0.222	0.416
		0.500	0.492	0.492	0.489	0.362	0.458	0.500
		0.750	0.500	0.500	0.500	0.482	0.499	0.500
100	0.044	0.000	0.044	0.044	0.044	0.045	0.044	0.044
		0.250	0.392	0.391	0.387	0.201	0.307	0.478
		0.500	0.500	0.500	0.499	0.446	0.494	0.500
		0.750	0.500	0.500	0.500	0.499	0.500	0.500

Table 4: Power of one-tailed t and sign tests for low, medium, and high kurtosis parent populations.

4 Discussion

Skewness in the sampling distribution of the t-statistic depends on the size and direction of the shift as well as the skewness coefficient of the parent population. In the case of a right-skewed parent population, a shift to the left will decrease skewness in the sampling distribution of the t-statistic, whereas a shift to the right will increase the skewness coefficient of the sampling distribution of the t-statistic. The power of the t-test isn't necessarily diminished in the absence of normality. For example, the t-test with a normal parent has higher power than with a $\chi^2(2)$ parent for small shifts to the right but lower power for larger shifts to the right. On the other hand, the t-test with a $\chi^2(2)$ parent has higher power than with a normal parent for small shifts to the left but lower power for larger shifts to the left.

The t-test is uniformly higher in power than the sign test in cases where the shift is in the opposite direction of the skewness in the parent population. We have shown the sensitivity of the sign test to changes in kurtosis for symmetric parent populations. For symmetric parent populations, the power of the sign test is low for parent populations with low kurtosis and high for parent populations with high kurtosis. Particularly pointing out that the power of the sign test is less than the power of the t-test for a low kurtosis parent population, especially for smaller shifts.

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